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Investigating the Formation of Extra-Galactic Radio Jets

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INVESTIGATING THE FORMATION OF EXTRA-GALACTIC RADIO JETS

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ABSTRACT

Jets of collimated particles are among the most intriguing characteristics of active galactic nuclei. In this paper two models of particle interactions in active galactic nuclei are used to explore a possible initial acceleration mechanism of the jets: charge separation. Model I ignores the effects of both collisions and electromagnetic forces and does not predict charge separation. Model II incorporates collisions and electromagnetic interactions, and, although it too predicts no charge separation, it provides a more physically sound model of active nucleus than does Model I.

BACKGROUND INFORMATION

Before discussing active galaxies, it is important to understand what is meant by the term *galaxy*. A typical galaxy, like our own Milky Way, is a collection of one hundred billion stars and interstellar dust held together by its own gravity. Normal and Active galaxies are distinguished by the properties of their centers or nuclei. Active galactic nuclei, or AGN, are brighter and more dense than the nuclei of normal galaxies. AGN also produce collimated jets of particles which can be up to millions of light years long; the nuclei of normal galaxies do not. The goal of this paper is to explain the formation of these particle jets. Because the jets originate in AGN, a better understanding of AGN should lead to a better understanding of the jets.

AGN are very bright. Most are brighter than an entire normal galaxy, such as the Milky Way. In AGN, however, brightness equivalent to that produced by an entire normal galaxy arises from a volume no larger than that of our solar system. How so small a region can shine more intensely than one hundred billion stars is not immediately obvious. Observations of the changing brightness of AGN can be used to impose a constraint on their size. The highest speed at which information can be transferred is **c**, the speed of light. If an object exhibits coherent brightness changes over a period of time, **t**, the size of the object is limited to the distance, **ct**, that light has traveled during time **t** (Shu). The brightness of some AGN varies over time periods as short as three hours. The size of these AGN is, therefore, limited to three light hours: half the size of our solar system. Because one hundred billion stars cannot fit in a space half the size of our solar system, the brightness of AGN must be accounted for by something other than stars.

The most widely accepted theoretical account of AGN is the accreting supermassive black hole model. Black holes form when a star reaches its critical density and collapses under its own gravity. A collapsing star contracts to a geometrical point of infinite density. This point, called the singularity, is the center of a black hole. The boundary of a black hole is the *Schwarzchild radius* or *event horizon*, which is the distance from the singularity at which the gravitational pull of a black hole becomes so strong that not even light can escape it. A black hole is said to be *supermassive* if it is many times more massive than our own sun. An *accreting* supermassive black hole is a supermassive black hole that is gaining matter from its surroundings.

Observations support the idea that supermassive black holes exist in AGN. The masses of galactic nuclei can be determined by observing the orbital velocity of material around them. The orbital velocity of matter in galaxies depends on the mass interior to it. Using the Hubble Space Telescope, high orbital velocities have been detected in a number of AGN. High orbital velocities imply large central mass. Evidence for dark, compact objects, probably supermassive black holes, with masses of $2.4\ (\pm\ 0.7) * 10^9$ and $1.5\ (\pm\ 0.6) * 10^9$ solar masses has been found in the active galaxies M87 (Harms et. al., 1994) and M84 (Bower et. al., 1998), respectively. These results have led researchers to conclude that typical AGN contain supermassive black holes.

Gas and dust particles surrounding the supermassive black hole in an AGN congregate preferentially in the equatorial plane perpendicular to the black hole's axis of rotation under the influence of its gravity. A qualitative understanding of this rotational preference can be gained by likening the process of orbital orientation to pizza making. As a pizza maker spins a ball of mass, in this case pizza dough, the mass from the original blob of dough becomes a disk distributed evenly about the axis of rotation. While chemical bonds are what hold a spinning disk of dough together, in an analogous way, the black hole's gravity binds particles into the shape of a disk in an active galaxy. This region is known as the *accretion disk* because matter builds up here before accreting onto the super massive black hole.

Within the framework of the accreting supermassive black hole model, one may begin to examine how the jets produced by AGN might form. No consensus yet exists regarding the acceleration mechanism responsible for the jets, but observed characteristics of the jets offer insight into their nature. The jets produce little to no light at optical wavelengths. Instead, they are observed by the radio-wavelength *sychrotron radiation* they emit. *Synchrotron radiation* is emitted by high-velocity charged particles as they spiral around strong magnetic field lines. A model of the jets should account for synchrotron radiation and the strong magnetic fields they exhibit.

The remaining sections of the thesis are devoted to examining the possibility of charge separation in the active nucleus and its viability as an acceleration mechanism for the jets. If charge separation indeed occurs in the disk, net currents comprised of orbiting charged particles would form. These currents would create magnetic and electric fields conducive to the formation of jets which emit synchrotron radiation.

PRELIMINARY ASSUMPTIONS FOR MODELS I & II

The assumptions explained below are valid for both of the following models of the accretion disk. These assumptions concern the type of particles present in the accretion disk and the radial boundaries of the disk within which the models will be evaluated.

Because approximately 90% of the universe is comprised of hydrogen, one can assume that the accreting material of AGN is comprised entirely of hydrogen.

To provide accreting material for the black hole, particles in the disk must experience some infall rate. If the infall rate is such that electrons fall toward the black hole faster than protons or vice versa, charge separation will develop. If sufficient charge separation is present, it could account for the jets. To determine whether charge separation in the disk is theoretically possible, boundaries within which to analyze the disk must first be selected.

The distance from the black hole at which hydrogen becomes ionized, or the ionization radius, $\mathbf{R}_i$, will serve as the outer boundary of the disk. $\mathbf{R}_i$, can be determined from observations. Using the Hubble Space Telescope, (Ferrarese et. al.,1996) have observed a disk of gas and dust in the center of NGC 4261, a typical active galaxy. The inner, ionized region of the disk has a radius, $\mathbf{R}_i >\sim 10^{17}$ m, a few hundred light years.

Because equations used in relativistic analysis are often more complicated than those apply will be used to evaluate models of the disk. The innermost radius of the non-relativistic region of the disk can be determined using the relativistic limit of orbital velocity of particles about the black hole. At velocities of about one-tenth the speed of light, the discrepancy between predictions made by classical physics and relativity theory become significant ($\geq$ 1% error). The equation for orbital velocity is $\mathbf{v}_{\mathbf{orbital}} = (\mathbf{GM/r})^{1/2}$, where **G** is the universal gravitation constant and **M** is the mass of the black hole, and **r** is the distance to the black hole. Setting the orbital velocity equal to one-tenth the speed of light, $(\mathbf{GM/r})^{1/2} = 0.1\mathbf{c}$, and solving for **r** yields the distance from the innermost non-relativistic orbit to the black hole, $\mathbf{R}_{\mathbf{rel}} >\sim 10^{14}$ m : approximately one hundred times the radius of our solar system.

Studying the disk within non-relativistic limits excludes part of its interior. It is instructive to know how much of the disk lies between the non-relativistic boundary and the black hole. The boundary of a black hole is defined in terms of the Schwarzchild radius, $\mathbf{R}_{\mathbf{sch}}$, where $\mathbf{R}_{\mathbf{sch}} = 2\mathbf{GM/c}^2$ (Shu). Thus, for the estimated mass of the black hole, $\mathbf{M} = 10^{39}$ kg,

$\mathbf{R}_{\mathbf{Sch}} >\sim 10^{12}$ m.

It is interesting to note that the non-relativistic region extends to within two orders of magnitude of the Schwarzchild radius, the ultimate general relativistic limit. With boundary conditions and preliminary assumptions established, it is now possible to construct simple models of the accretion disk and determine the degree of initial charge separation therein.

MODEL I

Infall rate of particles in the disk

In model I, the disk is treated as a two-dimensional collision-less plane that is coincident with the black hole's equatorial plane and electrically neutral about its periphery. One may assume that the particle density at the outer edge of the disk will be typical of the observed particle density in interstellar gas clouds: 20 to 25 particles per cubic centimeter (Miniati et al.). This low density suggests that particles on the outer edge of the accretion disk will experience very few collisions. If one assumes that positive and negative charges are distributed evenly in the region of initial ionization, electrical interactions can be ignored.

In this simplified model of the disk, the total energy of a particle is simply the sum of its kinetic and potential energies. For a given particle, the total energy, $\mathbf{E_p}$, can be expressed as

$$\mathbf{E_p} = {}^1/_2\mathbf{mv^2} - \mathbf{GMm/r}, \qquad (1)$$

where **m** and **v** denote the mass and velocity of the particle, respectively.

Under the influence of gravity, particles in the disk will be continuously accelerated. Accelerated charges radiate energy. The Larmor Formula (Jackson) describes the power, $\mathbf{P_\gamma}$, of a photon radiated by an accelerated charge:

$$\mathbf{P_\gamma} = {}^2/_3\mathbf{e^2/c^3}\,(\boldsymbol{a})^2\,, \qquad (2)$$

where $\boldsymbol{a}$ denotes the particle's acceleration, **e** denotes the fundamental charge, and **c** denotes the speed of light.

The principle of conservation of energy requires that the power gained by the radiated photon must equal the power lost by the particle emitting it. Equating the Larmor Power to the negative rate of change of the charged particle's total energy, one can describe the energy transfer from the particle to the photon per unit time:

$$\mathbf{P}_\gamma = -\, {}^{d}/_{dt}\, \mathbf{E}_p = -\, {}^{d}/_{dt}\, ({}^{1}/_{2}\mathbf{m}\mathbf{v}^2 + \mathbf{GMm/r}) \qquad (3)$$

Therefore:

$$({}^{2}/_{3})(\mathbf{e}^2/\mathbf{c}^3)(\boldsymbol{a})^2 = -\, {}^{d}/_{dt}\, ({}^{1}/_{2}\mathbf{m}\mathbf{v}^2 + \mathbf{GMm/r}) \qquad (4)$$

As particles lose energy by emitting radiation, their orbits will decay, and they will spiral inward toward the black hole. The infall rate of particles associated with radiative energy loss will be defined in terms of particle mass, **m**, and distance to the black hole, **r**. To achieve such an expression, equations that express the acceleration of particles in the disk as functions of **m** and **r** will be considered.

The only acceleration operating on particles in this model of the disk is gravitational acceleration, which can be expressed in terms of centripetal acceleration ($\boldsymbol{a} = \mathbf{a}_c$). Centripetal acceleration is equal to both of the following expressions by definition and by the universal law of gravitation, respectively:

$$\mathbf{a}_c = \mathbf{v}^2/\mathbf{r} = \mathbf{GM/r}^2. \qquad (5)$$

It follows that

$$\mathbf{v}^2 = \mathbf{GM/r}. \qquad (6)$$

Substituting (5) and (6) into equation (4) yields an equation in which the only variables are particle mass and radius:

$$ {}^{2}/_{3}(\mathbf{e}^2/\mathbf{c}^3)(\mathbf{GM}/\mathbf{r}^2)^2 = -{}^{1}/_{2}\mathbf{Gm}\,{}^{d}/_{dt}\,(\mathbf{M}/\mathbf{r} + 2\mathbf{M}/\mathbf{r}), \tag{7} $$

which simplifies to

$$ ({}^{4}/_{9})(\mathbf{e}^2/\mathbf{c}^3)((\mathbf{GM}^2)/(\mathbf{mr}^4)) = {}^{d}/_{dt}\,(\mathbf{M}/\mathbf{r}). \tag{8} $$

Differentiating equation (8) yields

$$ ({}^{4}/_{9})(\mathbf{e}^2/\mathbf{c}^3)((\mathbf{GM}^2)/(\mathbf{mr}^4)) = ({}^{dM}/_{dt})/\mathbf{r} - (\mathbf{M}/\mathbf{r}^2)({}^{dr}/_{dt}). \tag{9} $$

Solving equation (9) for ${}^{dr}/_{dt}$ results in the following expression:

$$ {}^{dr}/_{dt} = ({}^{dM}/_{dt})(\mathbf{r}/\mathbf{M}) - (({}^{4}/_{9})(\mathbf{GMe}^2/\mathbf{c}^3))/(\mathbf{mr}^2) \tag{10} $$

where ${}^{dr}/_{dt}$ is the rate of approach toward the black hole, or *infall rate* for a particle of mass **m**.

If the constant $(({}^{4}/_{9})(\mathbf{GMe}^2/\mathbf{c}^3))$, which has a value of approximately 10^{-25} (kg)m^3 s^{-1} is denoted as **C**, equation (10) becomes

$$ {}^{dr}/_{dt} = ({}^{dM}/_{dt})(\mathbf{r}/\mathbf{M}) - \mathbf{C}/\mathbf{mr}^2. \tag{11} $$

Results and Conclusions

The result of Model I states that the infall rate of a given particle in the disk depends on its mass and the rate at which the supermassive black hole gains mass. This infall rate must be negative, meaning the distance of matter in the disk to the black hole is decreasing, for accretion to occur. A non-negative infall rate, $^{dr}/_{dt} \geq 0$ implies expulsion of material or a static disk. To satisfy the condition $^{dr}/_{dt} < 0$, requires $(^{dM}/_{dt})(\mathbf{r}/\mathbf{M}) < \mathbf{C}/\mathbf{mr}^2$ or

$$(^{dM}/_{dt}) < \mathbf{CM}/(\mathbf{mr}^3). \qquad (12)$$

The accretion onto the black hole, $^{dM}/_{dt}$, is maximized when **m** and **r** are minimized. The minimum values of **m** and **r** considered in this model are $\mathbf{m}_{e-}$ (the mass of the electron) and $\mathbf{R}_{rel}$ (the innermost non-relativistic orbit), respectively. If one uses these minimum values and assumes a black hole mass of 10^9 solar masses, the maximum allowed value of $^{dM}/_{dt}$ is 10^2 kg s^{-1}.

The observed brightness of AGN provides a constraint on the minimum accretion rate of matter onto the supermassive black hole. The luminosity of a typical AGN, > 10^{37} W (Zeilik & Gregory), can be produced with an accretion rate of one tenth of a solar mass per year ($3*10^{21}$ kg s^{-1}), if one assumes that the black hole converts the gravitational potential energy of infalling matter into radiated energy with an efficiency of approximately ten percent. Unfortunately, this observationally imposed value is nineteen orders of magnitude greater than the maximum value of $^{dM}/_{dt}$ allowed by the model.

Even if accretion rates much, much less than 10^2 kg s^{-1} were possible, the model still would not yield a viable result. In such a case ($^{dM}/_{dt} << 10^2$ kg s^{-1}), the expression for $^{dr}/_{dt}$ reduces to

$$^{dr}/_{dt} \simeq \mathbf{C} / \mathbf{mr}^2 . \tag{13}$$

This result suggests that as a particle's orbital radius decreases, its infall rate increases. It also implies that, at least initially, an electron will fall toward the black hole approximately 1832 times faster than a proton beginning at the same radius. Because the mass of a proton is approximately 1832 times that of an electron, the ratio [$^{dr}/_{dt}$ {electron}] / [$^{dr}/_{dt}$ {proton}] will be approximately 1832. Because this result predicts charge separation, it could substantiate the idea that the jets of particles arising from AGN are driven by electric fields.

The infall rates of protons and electrons will be greatest at the inner boundary of the disk, $\mathbf{R}_{rel}$. If one substitutes the value $\mathbf{R}_{rel}$ into equation (13), the model predicts a maximum infall rate of approximately 10^{-25} ms^{-1} for a electron and approximately 10^{-28} ms^{-1} for a proton. Using this result and the number of seconds in a year, one can deduce that in the age of the universe (approximately 10^{10} years) electrons in the disk would have moved approximately $30*10^{-9}$ m toward the black hole, a few hundred times larger than the size of an atom, while protons would have moved only about $.03*10^{-9}$ m from their starting point, roughly the size of a single atom. The infall rate predicted by the model in the case of $^{dM}/_{dt} << 10^2$ kg s^{-1} is, therefore, effectively zero. Thus, the result obtained if one assumes a small accretion rate amounts to restating that the accretion rate is small, a circular argument.

The null result produced by Model I does lead to some useful information. It shows that radiative energy losses from acelerated charged particles, the only energy loss mechanism assumed in the model, will not create infall. Therefore, radiative losses can be ignored in future models.

MODEL II

The Collisional Force

The likelihood that charge separation occurs in the active nucleus can be more realistically assessed by incorporating collisional interactions into the model of the disk. Factors involved in analyzing collisional interactions among particles in the disk are relative particle size and speed and collision frequency. Knowledge of these quantities can be used to define an expression for the collisional force acting between particles.

The effective sizes of charged particles can be determined by comparing their electric potential and relative kinetic energies. The minimum separation of two particles with equal charge is determined by their relative velocity. Equating the kinetic and electrical potential energies of a two particle system yields

$$\tfrac{1}{2}(\mathbf{m}_1 + \mathbf{m}_2)(\mathbf{v}_{rel})^2 = \mathbf{k}\mathbf{q}_1\mathbf{q}_2/\mathbf{d}_{min} \quad (14)$$

where $\mathbf{m}_1$ and $\mathbf{m}_2$ are the masses of the particles, $\mathbf{v}_{rel}$ is their relative velocity, **k** is a constant with a magnitude of approximately $9*10^9$ NmC^{-2}, $\mathbf{q}_1$ and $\mathbf{q}_2$ are the charges of the particles, and $\mathbf{d}_{min}$ is the minimum distance between the particles. If one assumes that the two particles carry the fundamental charge, **e**, as in the case of protons or electrons, equation (14) simplifies to

$$\tfrac{1}{2}(\mathbf{m}_{p+} + \mathbf{m}_{e-})(\mathbf{v}_{rel})^2 = \mathbf{k}\mathbf{e}^2/\mathbf{d}_{min}, \quad (15)$$

where $\mathbf{m}_{P+}$ and $\mathbf{m}_{e-}$ represent the mass of a proton and the mass of an electron, respectively. If and electron and a proton, moving at speed $\mathbf{v}_{rel}$, come within a distance $\mathbf{d}_{min}$ of one another, they will engage in a coulomb collision. Solving equation (15) for $\mathbf{d}_{min}$, one finds

$$\mathbf{d}_{min} = 2\mathbf{k}\mathbf{e}^2/(\mathbf{m}_{P+} + \mathbf{m}_{e-})(\mathbf{v}_{rel})^2 \simeq 2\mathbf{k}\mathbf{e}^2/(\mathbf{m}_H)(\mathbf{v}_{rel})^2, \qquad (16)$$

where $\mathbf{m}_H$ is the mass of hydrogen. It is assumed that relative velocity, $\mathbf{v}_{rel}$, between particles in the disk is due to random thermal motions. From the Maxwell Boltzman distribution, the average relative speed of particles in an ideal gas is

$$\mathbf{v}_{MB} = \{[2\mathbf{k}_B\mathbf{T}(\mathbf{r})]/(\pi\ \mathbf{m})\}^{1/2}, \qquad (17)$$

where $\mathbf{k}_B$ is Boltzmann's constant and $\mathbf{T}(\mathbf{r})$ is temperature as a function of radius. The Maxwell Boltzman distribution describes the behavior of a homogenous gas of particles or molecules which have a mass of **m**. It, therefore, does not apply directly to a plasma (ionized gas) of two constituent particles, protons and electrons, which have different masses. It can be used to approximate the relative velocity of protons and electrons in the disk, however. According to equation (17), the relative particle velocity varies inversely with the square root of their mass. Thus, the predicted relative velocity of electrons is roughly 43 times the predicted velocity of protons at the same temperature ($\mathbf{m}_{e-}^{-1/2}$ is roughly 43 times greater than $\mathbf{m}_{p+}^{-1/2}$). In other words, with respect to the electrons, the protons are virtually standing still. Therefore, the relative velocity between all particles in the disk can be approximated by the relative velocity between electrons. One can define the relative velocity of the plasma in terms of relative electron velocity by replacing the **m** in equation (17) with $\mathbf{m}_{e-}$

$$\mathbf{v}_{rel} = \{[2\mathbf{k}_B\mathbf{T}(\mathbf{r})]/(\pi\ \mathbf{m}_{e-})\}^{1/2}, \qquad (18)$$

Understanding how collisions affect the position of protons and electrons in the disk, requires knowledge, not only of relative particle motion, but also of how often collisions occur. The mean-free path is the average distance a particle can travel through a surrounding medium without engaging in a collision. In a plasma of protons and electrons, the mean-free path is defined by the equation

$\lambda = ((2)^{1/2}\pi\, \mathbf{d}_{min}{}^2\, \mathbf{N/V})^{-1}$ (Halliday and Resnick), (19)

where **N/V** represents the number density of particles in the plasma.

How often particles collide, or their *collision frequency*, is an important quantity associated with the mean-free path. The relative velocity of colliding particles divided by their mean-free path gives yields their collision frequency, τ. The collision frequency is, therefore,

$\mathbf{v}_{rel}/\lambda = \tau$. (20)

The average force imparted by collisions is defined as the product of average particle momentum and collision frequency. The expression for the collisional force is thus

$\mathbf{F}_{col} = (\rho(\mathbf{r})\mathbf{V}\, \mathbf{v}_{rel})\tau$, (21)

where $\rho(\mathbf{r})$ is density as a function of radius, **V** is volume, and $\rho(\mathbf{r})\mathbf{V}$ represents the mass of a portion of the disk (the product of density and volume is mass). If one substitutes equation (20) into equation (21), the collision force can be written in terms of average relative velocity and mean free path

$\mathbf{F}_{col} = \rho(\mathbf{r})\mathbf{V}\, \mathbf{v}_{rel}(\mathbf{v}_{rel}/\lambda) = \rho(\mathbf{r})\mathbf{V}(\mathbf{v}_{rel})^2/\lambda$. (22)

If one substitutes equation (19) into equation (22), the formula for the collsional force can be rewritten

$$\mathbf{F}_{col} = \rho(\mathbf{r})\mathbf{V}(\mathbf{v}_{rel})^2[(2)^{1/2}\pi\, \mathbf{d}_{min}{}^2\, \mathbf{N/V}]. \quad (23)$$

After $\mathbf{d}_{min}$ is replaced with the expression in equation (16), equation (23) becomes

$$\mathbf{F}_{col} = \rho(\mathbf{r})\mathbf{V}(\mathbf{v}_{rel})^2[(2)^{1/2}\pi\, 4\{\mathbf{ke}^2/(\mathbf{m}_H)(\mathbf{v}_{rel})^2\}^2\, \mathbf{N/V}]$$

$$= \rho(\mathbf{r})\mathbf{V}[(2)^{1/2}\pi\, 4\{\mathbf{ke}^2/(\mathbf{m}_H)(\mathbf{v}_{rel})\}^2\, \mathbf{N/V}]. \quad (24)$$

If one substitutes $\rho(\mathbf{r})/\mathbf{m}$ for $\mathbf{N/V}$ and rewrites $\mathbf{v}_{rel}$ using formula (18), equation (24) can be recast

$$\rho(\mathbf{r})^2\, \mathbf{V}\{(2)^{3/2}\pi\, [\mathbf{ke}^2/(\mathbf{m}_H)]^2\, \{(\pi\, \mathbf{m}_{e-})/[\mathbf{k}_B\mathbf{T}(\mathbf{r})\mathbf{m}]\}\}, \quad (25)$$

where $\mathbf{m}$ is the mass of the particle with respect to which the collisional force is evaluated.

If the constant, $(2)^{3/2}\pi\, [\mathbf{ke}^2/(\mathbf{m}_H)]^2\, (\pi\, \mathbf{m}_{e-})$, which appears in equation (25) and has has a value of 10^{-31} kg m^6 s^{-4}, is replaced with α , the collisional force can be written as

$$\mathbf{F}_{col} = \alpha\rho(\mathbf{r})^2\, \mathbf{V}/[\mathbf{k}_B\mathbf{T}(\mathbf{r})\mathbf{m}]. \quad (26)$$

$\mathbf{F}_{col}$ is expressed, in equation (26), as an implicit function of the temperature and density of particles in the disk. Both temperature and density are written as implicit functions of radius; no expression has yet been designated for either term. An understanding of how $\mathbf{F}_{col}$ varies with radius, therefore, requires the derivation of formulas which express temperature and density as explicit functions of radius.

Temperature as a function of radius

If one assumes that the disk is *adiabatic* (negligible energy due to conduction, convection, or radiation is exchanged between particles in the disk),

$\mathbf{T}\,[\mathbf{V}\,(\mathbf{r})]^{\gamma-1} \simeq$ a constant

and, therefore,

$$\mathbf{T}_i\,[\mathbf{V}\,(\mathbf{R}_i)]^{\gamma-1} \simeq \mathbf{T}\,[\mathbf{V}\,(\mathbf{r})]^{\gamma-1} \tag{27}$$

where **T** represents temperature, $\mathbf{T}_i$ represents hydrogen's ionization temperature, **V (r)** is the volume of the disk at some distance, **r**, from the black hole, and **V** $(\mathbf{R}_i)$ is the volume of the disk bounded by $\mathbf{R}_i$. The symbol $\gamma = (\mathbf{2} + \nu)/\nu$, where ν is equal to the number of degrees of freedom per particle in the system. In this case $\nu = 3$ kinetic degrees of freedom (particles can move freely in all three directions in space). If $\nu = 3$, then $\gamma = {}^5/_3$ and, therefore, $\gamma - 1 = {}^2/_3$.

Temperature can now be expressed as a function of volume:

$$\mathbf{T}(\mathbf{r}) = \mathbf{T}_i\,(\mathbf{V}\,(\mathbf{R}_i)\,/\mathbf{V}\,(\mathbf{r}))^{2/3}. \tag{28}$$

T(r) can be expressed as an explicit function of **r** if **V (r)** is defined explicitly. The expression for the volume of the region of the disk analyzed in the model is that of an annulus with inner radius **r** and outer radius **r + dr**, such that $\mathbf{R}_i \geq \mathbf{r} \geq \mathbf{R}_{rel}$ and **dr** is an incremental element of radius. The differential volume, **dV**, of a region of the disk bounded by **r** and **r + dr** with thickness **z** is defined by the formula $\mathbf{dV} = \pi((\mathbf{r} + \mathbf{dr})^2 - \mathbf{r}^2)\mathbf{z}$, which reduces to $\mathbf{dV} = 2\pi\mathbf{rdrz}$ if the $(\mathbf{dr})^2$ term in the binomial expansion is ignored. Integration of **dV** from $\mathbf{R}_{rel}$ to **r** yields the following equation

$$\mathbf{V}(\mathbf{r}) - \mathbf{V}(\mathbf{R}_{rel}) = \pi\mathbf{z}(\mathbf{r}^2 - \mathbf{R}_{rel}{}^2). \tag{29}$$

If one recognizes that $\mathbf{V}(\mathbf{R}_{rel}) = \pi\mathbf{z}(\mathbf{R}_{rel})^2$, equation (29) simplifies to the expression $\mathbf{V} = \pi\mathbf{r}^2\mathbf{z}$.

$\mathbf{T}(\mathbf{r})$ can now be expressed explicitly as

$$\mathbf{T}(\mathbf{r}) = \mathbf{T}_i([\pi\mathbf{R}_i{}^2\mathbf{z}_i] / [\pi\mathbf{r}^2\mathbf{z}])^{2/3}. \tag{30}$$

If one assumes that the gravitational pressure that flattens the disk is in the same balance with the pressure of its heating gas throughout, the thickness of the disk will be constant (as is the thickness of the Milky Way's galactic disk). If the thickness of the accretion disk is assumed to be constant, $\mathbf{z}_i = \mathbf{z}$, and equation (30) simplifies to

$$\mathbf{T}(\mathbf{r}) = \mathbf{T}_i(\mathbf{R}_i/\mathbf{r})^{4/3}. \tag{31}$$

Equation (31) predicts that the temperature of the disk will increase as the distance to the black hole decreases. This means that matter nearest the center of the nucleus produces the most energy, and that is consistent with observation. Using equation (31), the collisional force simplifies to the form

$$\mathbf{F}_{col} = \beta[\rho(\mathbf{r})^2\mathbf{V}(\mathbf{r}/\mathbf{R}_i)^{4/3}]/\mathbf{m}, \tag{32}$$

where β is a constant with a value of 10^{-13} m^4 s^{-2}. The collisional force, as expressed in equation (32), is an explicit function of $\mathbf{r}$ and $\rho(\mathbf{r})$. In order to understand the effect of collisions on particle motion as a function of radius, an expression for the density as a function of radius, $\rho(\mathbf{r})$, must be derived.

Density as a function of radius

An expression for the density as a function of radius can be derived using the virial theorem -- which states that the total energy of a statistically static system is equal to half of its gravitational potential energy (Shu). A statistically static system is one that may involve dynamic interactions but as a whole exhibits no distinguishable differences over time. It is assumed that the accretion disk is a statistically steady object. The total energy of particles in the accretion disk is equal to the sum of the work done by collisional and gravitational forces. By the virial theorem, this sum is equal to one half of the gravitational potential energy,

$$\Sigma(\mathbf{F}_{col} + \mathbf{F}_G)\cdot\mathbf{dr} = {}^1\!/_2\, \mathbf{E}_G. \tag{33}$$

Equation (33) can be rewritten in differential form:

$$\beta[\rho(\mathbf{r})^2\, \mathbf{V}(\mathbf{r}/\mathbf{R}_i)^{4/3}]/\mathbf{m}\} + \mathbf{GM}\, \rho(\mathbf{r})\mathbf{V}/\mathbf{r}^2 = {}^1\!/_2\, \mathbf{GM}\, {}^d\!/_{dr}(\rho(\mathbf{r})\mathbf{V}/\mathbf{r}). \tag{34}$$

The expression for **V** derived in the last section was $\mathbf{V} = \pi\mathbf{z}\mathbf{r}^2$. Substitution of this expression into equation (34) yields

$$\beta[\rho(\mathbf{r})^2\, \pi\mathbf{z}\mathbf{r}^2(\mathbf{r}/\mathbf{R}_i)^{4/3}]/\mathbf{m}\} + \mathbf{GM}\pi\mathbf{z}\rho(\mathbf{r})\mathbf{r}^2/\mathbf{r}^2 = {}^1\!/_2\, \mathbf{GM}\pi\mathbf{z}\, {}^d\!/_{dr}(\rho(\mathbf{r})\mathbf{r}^2/\mathbf{r}),$$

which, after dividing through by $\mathbf{GM}\pi\mathbf{z}$ and combining like terms, simplifies to

$$\rho(\mathbf{r})^2\mathbf{r}^{10/3}\{\beta/\mathbf{GMm}(\mathbf{R}_i)^{4/3}\} + \rho(\mathbf{r}) = {}^1\!/_2\, {}^d\!/_{dr}(\rho(\mathbf{r})\mathbf{r}). \tag{35}$$

Differentiation of the right-hand side of equation (35) leads to the expression

$$\rho(\mathbf{r})^2\mathbf{r}^{10/3}\{\mathbf{GMm}(\mathbf{R}_i)^{4/3}\} + \rho(\mathbf{r}) = {}^1\!/_2\, \{{}^d\!/_{dr}[\rho(\mathbf{r})]\mathbf{r} + \rho(\mathbf{r})\}. \tag{36}$$

After multiplying through by 2 and replacing the constant, $\{2\,\beta/\mathbf{GM}(\mathbf{R}_i)^{4/3}\}$, which has a value of $10^{-66}\,m^{-1/3}\,kg^{-1}$,with the symbol ζ, equation (36) becomes

$$\zeta\rho(\mathbf{r})^2\mathbf{r}^{10/3}/\,\mathbf{m} + 2\rho(\mathbf{r}) = \{{}^d\!/_{dr}[\rho(\mathbf{r})]\mathbf{r} + \rho(\mathbf{r})\}. \tag{37}$$

After like terms are combined, equation (37) can be rewritten

$$\zeta\rho(\mathbf{r})^2\mathbf{r}^{10/3}/\,\mathbf{m} + \rho(\mathbf{r}) = {}^{d}/_{dr}[\rho(\mathbf{r})]\mathbf{r}. \tag{38}$$

After dividing through by the highest power of $\rho(\mathbf{r})$, equation (38) becomes,

$$\zeta\mathbf{r}^{10/3}/\,\mathbf{m} + \rho(\mathbf{r})^{-1} = \rho(\mathbf{r})^{-2}\,{}^{d}/_{dr}[\rho(\mathbf{r})]\mathbf{r}. \tag{39}$$

Equation (39) can now be rewritten

$$-\zeta\mathbf{r}^{10/3}/\,\mathbf{m} = {}^{d}/_{dr}[\rho(\mathbf{r})^{-1}\mathbf{r}]. \tag{40}$$

Integrating equation (40) and solving for $\rho(\mathbf{r})$ yields

$$\rho(\mathbf{r}) = \mathbf{Am}/\mathbf{r}^{10/3}\,. \tag{41}$$

The arbitrary constant of integration, A, can be evaluated using the observed density of particles in the disk at $\mathbf{R_i}$:

$\rho(\mathbf{R_i}) = \mathbf{Am}/\mathbf{R_i}^{10/3} = \rho_i = 10^8$ particles per m^3.

Algebraic manipulation yields

$A = \mathbf{R_i}^{10/3}\rho_i\,/\mathbf{m}.$

Equation (41) can now be expressed explicitly:

$$\rho(\mathbf{r}) = \rho_i(\mathbf{R_i}\,/\mathbf{r})^{10/3}\,. \tag{42}$$

Equation (42) has the same form as equation (31), which describes the variation of temperature as a function of radius. Equation (42) predicts that the density of particles in the accretion disk increases rapidly as **r** decreases. This is consistent with observation.

Now that $\rho(\mathbf{r})$ has been derived, $\mathbf{F_{col}}$ can be described as an explicit function of radius

$$\mathbf{F_{col}} = \kappa\{(\mathbf{R_i}/\mathbf{r})^{10/3}\}/\mathbf{m}, \tag{43}$$

where κ is a constant with a value of 10^{-66} kgN.

The expression for the collisional force is of the same form as the expressions for $\rho(\mathbf{r})$ and $T(\mathbf{r})$. Its magnitude increases as distance to the black hole decreases. This result is consistent with the density structure of the disk that $\rho(\mathbf{r})$ predicts: density increases as **r** decreases, thereby making collisions more probable as particles approach the black hole. The expression for also predicts charge separation. Unfortunately, its maximum value is eleven orders of magnitude smaller than the minimum value of the only other force considered in this model, the gravitational force. Because gravitation, which does not induce charge separation, is the dominant influence on particle motion, as a whole, Model II does not predict charge separation.

Results and Conclusions

Model II investigates the effect of collisions on the movement of particles in the disk. An equation for the average collisional force is derived in terms of the relative particle velocity and the mean-free path, the distance a particle can travel before engaging in a collision. To evaluate the mean-free path and relative particle velocity, expressions are derived for the temperature and density as a function of radius, $\mathbf{T}(\mathbf{r})$ and $\rho(\mathbf{r})$ respectively. These expressions predict that both temperature and density increase as radius decreases. This is consistent with observation.

The derivations of $\mathbf{T}(\mathbf{r})$ and $\rho(\mathbf{r})$ allow one to express the collsional force as an explicit function of radius. The equation for the collsional force, $\mathbf{F}_{col}$, predicts that the strength of collisional interaction increases as $\mathbf{r}$ decreases. This is consistent with a disk in which density increases as $\mathbf{r}$ decreases, the result one obtains from $\rho(\mathbf{r})$. However, at all radii, the particle motion is dominated by the force of gravity. The net result is an insignificant degree of charge separation.

SUMMARY OF RESULTS AND FUTURE WORK

Approximating the disk as a non-collisional, evenly charged plane in Model I produced results for particle infall rate that were algebraically well-defined, but numerically inconsistent with observation. Model I also fails to predict charge separation in AGN. It does, however, show that radiative energy losses from accelerated charge particles will not create infall. This result is important, because it allows one to ignore the effects of radiative losses in future models of the disk.

Model II incorporates collisions and electromagnetic interactions. It provides a more physically sound model of the disk. The major equations derived in Model II predict that temperature, $\mathbf{T}(\mathbf{r})$, density, $\rho(\mathbf{r})$, and the strength of collisional interactions, $\mathbf{F}_{col}$, increase as distance to the black hole decreases. These results are consistent with observation. Unfortunately, despite its many strong points, Model II also fails to predict charge separation.

Although this is the same as the final result obtained in the first model, as a whole, the strength of Model II far surpasses that of Model I. Only one major result is derived in Model I and it is shown to be inconsistent with observation. The results of Model II, on the other hand, are consistent with what is observed in nature. Including collisions and disregarding radiative losses as influences upon particle motion seems to have made Model II much more physically sound than Model I.

Even if either model of the disk would have predicted charge separation, more work would still need to be done. An electric field produced by a charge separation would create a singly charged jet. Inter-repulsions of like charges in a singly charged jet could compromise its stability if particles were too concentrated and/or moving too slowly. Observations of jets emitted by AGN show that they are stable and collimated over lengths of up to millions of light years, however. Therefore, even if it is shown to be mathematically probable, an electric field can be treated as a realistic means of particle acceleration only if the density of like charges in the jet is low enough to avoid instability or if charge balance is somehow restored within the jet.

Future work on this project could be devoted to assessing the viability of electric-field and/or otherwise-induced particle acceleration as an initiation mechanism for a *stable* jet. The gradual incorporation of new ideas into models of the disk has proven to be beneficial, and, if continued, work on this project would proceed in this manner. Points of emphasis for future work include the possible amplification of an existing magnetic field in the medium; radiation transfer from heating particles as an energy loss mechanism; Eulerian hydrodynamics to more accurately model the fluid character of the gas in the disk; and analysis of particle motion in the relativistic regime of the disk.

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