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# AN INVESTIGATION OF THE ACCURACY OF PARALLEL ANALYSIS FOR DETERMINING THE NUMBER OF FACTORS IN A FACTOR ANALYSIS

A Capstone Project Presented in Partial Fulfillment

of the Requirements for the Degree Bachelor of Psychological Science

with Honors College Graduate Distinction at

Western Kentucky Univeristy

By

Mandy M. Matsumoto

May 2017

*****

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I dedicate this thesis to my husband, David Matsumoto, who has been a great source of support and strength.

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## ABSTRACT

Exploratory factor analysis is an analytic technique used to determine the number of factors in a set of data (usually items on a questionnaire) for which the factor structure has not been previously analyzed. Parallel analysis (PA) is a technique used to determine the number of factors in a factor analysis. There are a number of factors that affect the results of a PA: the choice of the eigenvalue percentile, the strength of the factor loadings, the number of variables, and the sample size of the study. Although PA is the most accurate method to date to determine which factors are valid, there is still room for improvement. One area of PA not yet examined concerns the degree to which the actual eigenvalue exceeds the random data-based eigenvalue. All methods to date accept that any amount of difference, regardless of how trivial, between the two eigenvalues is enough to satisfy the criterion for a meaningful eigenvalue. However, a more prudent course may be the use of a greater margin than a simple absolute difference between the two eigenvalues. This research examines the accuracy of the simple difference versus the 10% standard. This standard will be the difference of 10% or more between the actual eigenvalue and the parallel analysis eigenvalue. Also examined are the efficacy of this standard in conjunction with the 50th, 90th, 95th and 99th percentiles. Results indicated that the 99th percentile, in conjunction with the 10% standard, is a more accurate method for determining accuracy when using parallel analysis to determine valid factors in a common factor analysis.

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# INTRODUCTION

It is important in research to ensure that the components being studied are measured accurately. When a questionnaire is given to participants, the accuracy with which each question relates specifically to the intended factor being studied is paramount in order for research to be valid. Through factor analysis, researchers can determine the number of factors, the relations of the items to these factors, and the relations of these factors to each other. There are two techniques available to determine the factor structure of a measurement instrument: exploratory factor analysis and confirmatory factor analysis.

Exploratory factor analysis is used to identify the factor structure of an instrument that has not previously been analyzed (i.e., when the researcher possesses very little information regarding the factor structure). As an example, an exploratory factor analysis may be run to determine the factor structure of a questionnaire designed to measure customer satisfaction. The questions listed in such a questionnaire may pertain to different aspects of customer satisfaction. Although each aspect may make up the whole, they are distinctly different parts. There may be questions in regards to the sales process, product satisfaction, and accessibility to the store. It is important to know which aspects of customer satisfaction are being assessed. It is also important to know if any question bears weight with more than one category, or aspect of customer satisfaction. This information will make interpreting the results of the questionnaire easier.

Confirmatory factor analysis is used to test a hypothesized factor structure. Confirmatory factor analysis often follows an initial exploratory factor analysis, and is performed on a new dataset with the goal of testing the factor structure suggested by the

exploratory analysis. Confirmatory factor analysis can also be used to test competing hypothesized factor models against each other with the goal of identifying the model that better fits the data.

Principal Component Analysis (PCA) is a simplified form of factor analysis. "The primary purpose of PCA is data reduction based on a straightforward mathematical transformation of a covariance or correlation mix" (Glorfeld, 1995). The theoretical model of PCA differs from factor analysis in two ways. First, the common factor model is a causal model indicating relationships between test items and the factors that cause the responses to these items, whereas PCA is simply a procedure to reduce a large number of variables down to a smaller number of variables. Second, in PCA the total variance of each variable is factored, whereas in CFA only the common variance (i.e., variance shared among items) is factored. Because the amount of common variance must be estimated, factoring only the common variance makes the model for CFA more complex than that used for PCA (Glorfeld, 1995). In the common factor model, each item can be divided into a common and a unique component. Given the usual standardization of variables in factor analysis, these two components sum to 1.0. The common component is due to the action of the common factors, whereas the unique component is unrelated to the common factors as well as the other unique components. The percent of a variable's variance that is common variance is called communality. The commonality between variables can be seen through the factor loadings.

Factor loadings show how well a particular variable loads onto a given factor. Higher factor loadings indicate stronger relationships. Factors with large loadings onto many items are larger factors than those with smaller loadings or similar sized loadings

on fewer items. The sum of the squared loadings across items indicates a factor's size and is called eigenvalue. An eigenvalue is also described as the variance accounted for by a factor. The first factor extracted will have the largest eigenvalue (i.e., have the largest factor loadings) with the remaining factors declining thereafter.

Common Factor Analysis and Principal Component Analysis are similar in many ways. They both use the variance accounted for by a factor (i.e., eigenvalue) to determine which components or factors are pertinent. Both procedures can be used to serve the purpose of variable reduction (although PCA is more directly suited for such a purpose). If the communalities are high, the results of the two procedures will be similar. However, due to CFA computing only the common variance, accuracy in factor determination can vary greatly depending on the methodology.

**Determining the Number of Factors**

There are a variety of methods for determining the number of factors in a factor analysis; however, none are perfect. A classic method of choosing factors of significance in a factor analysis is to use the eigenvalue greater than 1.0 test. This method has its origins in PCA and states that any factor that has an eigenvalue greater than 1.0 is a factor that is larger than any individual item should be retained (recall that all items are standardized and have a total variance of 1.0). The problem with this method is that it often overfactors (i.e., retains too many factors). There may be an eigenvalue that is just slightly over the 1.0 mark, with a value of 1.03, whereas another eigenvalue may be just slightly under 1.0, with a value of 0.997. The eigenvalue-greater-than-one rule would surmise that the former factor has significance while the latter does not. Yet both values are almost equal. Moreover, the factor retained is a rather weak factor, given that it is

only trivially larger than any one item. This flat rate method does not allow for any flexibility in regards to which factors are valid and which are not. Turner (1998) points out that drawing a line "rigidly at 1.0 is arbitrary" (pg. 542). In addition, this approach doesn't translate well to CFA, where only the common variance (not total variance) is factored. (Factorization of the common variance results in smaller eigenvalues than is found when the total variance is factored.)

The scree test is another method used to determine the number of factors in an exploratory factor analysis. The scree test sets the number of factors by a determination of the last eigenvalue that is substantially larger than the remaining eigenvalues. The logic of the test is as follows: On a *k* item test, the $k^{th}$ factor is, by definition, meaningless. (If all factors were of equal size, the $k^{th}$ factor would be equal to an individual item in magnitude. Given that each factor is smaller than the preceding factor, the $k^{th}$ factor is far less than an individual item in magnitude.) Any factor that is only trivially larger than the $k^{th}$ factor is, by extension, similarly meaningless. Only the factors that have a substantial increase in eigenvalue are considered valid. This is an easy method to use when there is a marked difference between the first few eigenvalues and the remaining eigenvalues. However, the clarity of the scree test begins to wane when there is not a marked difference between them. Such a situation could cause underfactoring and lead to errors in the conclusions researchers find.

Unfortunately, neither of the previously mentioned methods for determining the number of factors offer acceptable accuracy. In a comprehensive simulation study, Zwick and Velicer (1986) compared the various factor retention methods for accuracy. They found that the eigenvalue greater than 1.0 rule was only accurate 22% of the time. When

it was wrong, there was a tendency for overextraction. The Scree test was found to be accurate around 57% of the time. They found that when it was inaccurate, it had a high tendency for overextraction. (Zwick & Velicer, 1986)

One alternative to the aforementioned approaches is parallel analysis which was proposed by Horn (1965) as a method to improve the accuracy of determining the proper number of factors. Horn developed a process where random data is used to create eigenvalues with the same parameters of the actual variables. He "proposed that a number of correlated matrices of *p* uncorrelated random normal variables and a sample size equal to *n*, where *p* and *n* were the same as the corresponding entries in the data set under study, be constructed and their eigenvalues be averaged" (Glorfeld, 1995, p. 5). This process compares eigenvalues generated from an analysis of random data to the actual eigenvalues obtained from real data of equivalent size to determine the number of meaningful factors in a common factor analysis. A factor is considered valid if the actual eigenvalue exceeds the eigenvalue generated from the random dataset.

There has been much discussion on the accuracy of parallel analysis and how it should be used. Many authors, such as Weng and Cheng (2005, pg. 699), have pointed to the tendency for parallel analysis to overextract, in particular with smaller samples or when eigenvalues from actual data are compared to the mean (or $50^{th}$ percentile) eigenvalue from random data. Zwick and Velicer (1986) reported the overextraction of PA approximately 66% of the time when using the mean eigenvalue. They found that this method tended to extract "poorly defined factors with no high loadings on any variables" (Glorfeld, 1995, p. 380). For this reason, they suggested that PA be used with another method that had a tendency to underextract. Since their research many other researchers

have attempted to develop a more accurate way of applying PA in factor analysis. Glorfeld (1995) found that by modifying PA to adhere to $\alpha = .05$, or .01 (i.e., use of the $95^{th}$ or $99^{th}$ percentile instead of the $50^{th}$ percentile when averaging the results from the analyses of random data), the tendency for overextraction would be reduced. Weng and Cheng (2005) confirmed Glorfeld's finding in their research, stating:

> The results of the two-factor model followed the general conclusions from the one-factor model that parallel analysis correctly determined the number of factors and analysis using the $95^{th}$- and the $99^{th}$-percentile eigenvalues of the criteria that outperformed using mean eigenvalues as the basis for comparison. Moreover, when parallel analysis gave an incorrect number of factors, it was more likely to extract too many factors than to extract too few. The likelihood that the excellent performance of parallel analysis or the $95^{th}$- and $99^{th}$-percentile eigenvalues comes at the expense of underextraction is minimal. (pg. 711)

Glorfeld also found that parallel analysis does not rely on a normal distribution, but that, when modified, it will work with any distribution. Since most applications of parallel analysis are done through a variety of distributional settings, this distributional flexibility would allow it to be used in a variety of research settings.

In Horn's (1965) original study on parallel analysis, he found that when comparing the accuracy of the eigenvalue-greater-than-one rule (GK rule) to that of parallel analysis, the GK rule significantly overfactored in comparison. Zwick and Velicer (1986) compared the accuracy of parallel analysis to the GK rule, Bartlett's test, the scree test and Velicer's MAP. In their study that found that PA performed better than all the other factor retention methods.

More recently, Green, Thompson, Levy, and Lo (2015) proposed a modified version of parallel analysis. In their revision of PA, the $k^{th}$ eigenvalue is compared in the sample data and the random generated data. This varies from traditional PA, where the accumulative sample data is compared to the accumulative random data. In their study, they examined of PA using the 95$^{th}$ percentile in comparision with principal axis factoring (PAF). Their comparison of the two methods showed that, "traditional PA method using PAF and the 95$^{th}$ percentile rule (referred to as T-PA in this article) generally performed quite well in conditions except those with highly correlated factors" (p. 431). Given the similarity of results between the two methods, the focus of this study will concern traditional PA.

Based on the literature review so far, it seems that an important determinant of the accuracy of parallel analysis is the eigenvalue criterion. Crawford et al. (2010) examined the difference in factor determinations between the mean eigenvalue and the 95$^{th}$ percentile eigenvalue criterion. They found that in conditions with no underlying factors, maintaining a 95$^{th}$ percentile criterion correctly identified the lack of factors with an accuracy level of approximately 95%, whereas using the mean as a criterion correctly identified no factors approximately 53% of the time. When there are factors to be determined, the accuracy of the 95$^{th}$ percentile and the mean percentile begin to vary. Although the mean will be higher when there are up to three factors, the 95$^{th}$ percentile tended to be more accurate when there were more factors.

Turner (1998) argued that a revised parallel analysis where the 95$^{th}$ percentile is utilized instead of the 50$^{th}$ percentile had a higher tendency to underextract and would cause researchers to be at a higher risk for Type I errors. Turner stated that "the size of

noise eigenvalues depends on the presence of real common factors, the size of the real common factors, the pattern of structure coefficients of the common factors on the variables, and the distribution that make up the variables" (pg. 564). He added that although Glorfeld (1995) claimed that the 95$^{th}$ percentile eigenvalues would be more accurate, research by Zwick & Velicer (1986) show that using the mean eigenvalue tends to overextract only 5% of the time. Turner's explanation for this finding is twofold in that PA tends to "overestimate the size of noise eigenvalues when a real common factor is present and consequently underestimate the number of real factors, and traditional PA is usually based on the 50$^{th}$ percentile of completely random eigenvalues and should overestimate the number of values" (pg. 565). He concluded that due to the multiple aspects of parallel analysis affecting its accuracy (sample size, number of items, structure patterns, and size of known common factors) parallel analysis should be used in conjunction with other techniques to ensure accuracy of factor determination (Turner, 1998).

**The Present Study**

The previous research indicates that a number of factors affect the accuracy of factor extraction when using parallel analysis: the choice of eigenvalue percentile, the strength of the factor loadings, the number of variables, and the sample size of the study. Although PA is the most accurate method to date to determine which factors are valid, there is still room for improvement. One area of PA not yet examined concerns the degree to which the actual eigenvalue exceeds the eigenvalue derived from random data. All methods to date accept that any amount of difference, regardless of how trivial, between the two eigenvalues is enough to satisfy the criterion for a meaningful

eigenvalue. However, a more prudent course may be the use of a greater margin than a simple absolute difference between the two eigenvalues. It is proposed that a 10% standard may provide greater accuracy. This study will examine the accuracy of the simple difference versus a 10% standard. The efficacy of this standard will be examined in conjunction with the 50th, 90th, and 95th percentiles.

*Hypothesis*: Differences in the eigenvalue criterion (simulated eigenvalue standard: 50th, 90th, 95th, or 99th percentile; threshold: first real eigenvalue greater than simulated eigenvalue versus first real eigenvalue 10% greater than simulated eigenvalue) will lead to differences in the accuracy of the identified factor structure.

This study utilizes a Monte Carlo research model to test this hypothesis. Monte Carlo designs allow researchers the means to generate large datasets with known parameters and then to test hypotheses using these datasets. Two variables were manipulated for this study: the sample size for the factor analysis (120, a 10:1 ratio, or 240, a 20:1 ratio) and the number of iterations for the parallel analysis (200 or 500). For each of these four conditions, the following decision rules were investigated: the choice of eigenvalue from the Monte Carlo replications (the eigenvalue at the 50th, 90th, 95th, or 99th percentile) and the threshold by which the real eigenvalue must exceed the simulated eigenvalue for a given factor to be accepted (any difference versus a 10% difference).

# METHOD

## Population Generation

A dataset, representing a hypothetical population, was generated and consisted of 1,000,000 cases with scores on 12 variables. Following the factor structure from Glorfeld (1995), the variables in the population dataset were set so that they correlate in a manner consistent with a correlation matrix designed to produce a two factor structure. Each of the two factors was defined by four variables. In addition, two more variables had weak cross-loadings on the two factors, and two final variables did not load on any factors. The population correlation matrix is listed in Appendix A.

## Procedure

The experiment was conducted with the following procedure:

1. A sample of 120 or 240 cases was randomly selected from the population.
2. An exploratory factor analysis (common factor model) was performed on the sample data.
3. A parallel analysis (200 or 500 replications) was conducted on a sample of random data of the same size with the same number variables.
4. The eigenvalues (at either the $50^{th}$, $90^{th}$, $95^{th}$, or $99^{th}$ percentile) from the PA were compared to the eigenvalues obtained from the factor analysis of the sample data.
5. Based on the threshold criterion (any difference or 10% difference), the number of factors were determined for that sample by identifying the largest factor number (e.g., $2^{nd}$ factor, $3^{rd}$ factor, etc.) with a positive eigenvalue greater than the corresponding PA eigenvalue. In the event that a lower factor number was not greater than its corresponding PA eigenvalue, and a larger factor number was greater than its

corresponding PA eigenvalue (e.g., the eigenvalue for the 3rd factor was not greater than its PA eigenvalue, but the 4th factor was greater), all factors after the first non-greater factor were ignored (i.e., the 4th factor in the previous example would be ignored).

6. The process described in Steps 1-6 was repeated 1000 times, and the results was compared with the known population factor structure (i.e., two factors).

# RESULTS

The percent of cases in which the selection technique resulted in two factors (the actual number of factors in the population dataset) was computed for each condition (listed in Tables 1-4). An inspection of these results indicates two trends. First, across all four conditions the use of the eigenvalue at the 99$^{th}$ percentile was approximately equal to or clearly superior to 95$^{th}$ percentile value and was superior to all other values. Second, across all four conditions the use of a more stringent 10% threshold was approximately equal to or clearly superior to a simple difference between the eigenvalues. Thus, there was no instance in which any eigenvalue standard *less* than the 99$^{th}$ percentile clearly outperformed the 99$^{th}$ percentile, and there was no instance in which the simple absolute difference clearly outperformed the 10% margin of difference; however, there were many instances in which the converse was true. In summary, the most stringent eigenvalue standard appear to be the most effective eigenvalue standard.

With a smaller sample size, the accuracy of the 10% standard is approximately equivalent to the simple absolute difference at the 99$^{th}$ percentile level. However, the 10% standard does exceed the simple absolute difference when using the 95$^{th}$, 90$^{th}$ or 50$^{th}$ percentile. At the 50$^{th}$ percentile, the accuracy with or without the 10% standard is subtaintially poor that it should not be used as a method to determine valid factors. However, with a larger sample size, the accuracy of the 10% standard exceeds the simple absolute difference in all percentiles. In summary, the 99$^{th}$ percentile plus 10% standard is most accurate or nearly as accurate (within three percentage points) as the best rule across all conditions.

Table 1. Factor Determination Accuracy Percentages of Smaller Population with Higher PA Interations

| Condition | 99th | 95th | 90th | 50th |
|---|---|---|---|---|
| Any Difference | 76.3% | 64.2% | 53.3% | 19.4% |
| 10% Difference | 74.0% | 75.3% | 71.6% | 34.4% |

*Note*: Sample size = 120, number of iterations per PA = 500

Table 2. Factor Determination Accuracy Percentages of Smaller Population with Lower PA Interations

| Condition | 99th | 95th | 90th | 50th |
|---|---|---|---|---|
| Any Difference | 75.7% | 58.7% | 50.2% | 19.0% |
| 10% Difference | 73.2% | 75.9% | 69.1% | 33.1% |

*Note*: Sample size= 120, number of iterations per PA = 200

Table 3. Factor Determination Accuracy Percentages of Larger Population with Higher PA Interations

| Condition | 99th | 95th | 90th | 50th |
|---|---|---|---|---|
| Any Difference | 66.7% | 50.3% | 41.3% | 12.1% |
| 10% Difference | 83.1% | 67.4% | 59.7% | 26.6% |

*Note*: Sample size= 240, number of iterations per PA = 500

Table 4. Factor Determination Accuracy Percentages of Larger Population with Lower PA Iterations

| Condition | 99th | 95th | 90th | 50th |
|---|---|---|---|---|
| Any Difference | 70.4% | 52.2% | 42.0% | 13.4% |
| 10% Difference | 85.7% | 71.2% | 62.3% | 26.1% |

*Note*: Sample size= 240, number of iterations per PA = 200

## DISCUSSION

Utilizing the 10% standard increases accuracy with nearly all percentiles. (Smaller sample sizes with the 99% percentile were within three percentage points of accuracy in regard to absolute difference versus the 10% standard.) This accuracy increases with larger sample sizes. Using the mean percentile originally proposed by Horn (1965) shows the lowest accuracy rates, even with the 10% standard. When applying the 10% standard in conjunction with the 90th percentile to PA showed a marked increase in accuracy for factor determination. There was a 36.2% difference in accuracy when using the 90th percentile as opposed to the mean with a larger population. Accuracy continued to increase when applying the 10% standard to higher percentiles for PA. The highest accuracy rate was attained when applying the 10% standard in conjunction with the 99th percentile where N = 240, at 85.7%, a difference of 59.6% from when applying the 10% standard to the mean percentile.

### Further Research

This study used a population correlation matrix with Glorfeld's (1995) two-factor structure wherein each factor is defined by four variables. Glorfeld's correlation matrix

was designed to test overextraction. Thus, it is not surprising that the most stringent standard ($99^{th}$ percentile plus 10% margin) consistently produced the best results in this study. Research based on other factor structure models should be executed to further determine the accuracy of the 10% standard in conjunction with the $99^{th}$ percentile eigenvalue with particular attention to correlation matrices likely to produce underextraction.

**Conclusion**

By applying the 10% standard with a the $99^{th}$ percentile eigenvalue in PA, the accuracy of factor determination is increased as compared to other PA standards. This satisfies the recommendations of Glorfeld (1995) as regards a higher percentile than the mean when using PA in factor determination. Future research should explore the accuracy of this standard in datasets likely to produce underextraction as well as datasets with larger numbers of variables and factors.

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## APPENDI: CORRELATION MATRIX AND FREQUENCY TABLES

Appendix A:
Population Correlation Matrix

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0 | | | | | | | | | | | |
| 2 | .25 | 1.0 | | | | | | | | | | |
| 3 | .25 | .25 | 1.0 | | | | | | | | | |
| 4 | .25 | .25 | .25 | 1.0 | | | | | | | | |
| 5 | .04 | 0.0 | 0.0 | 0.0 | 1.0 | | | | | | | |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | .25 | 1.0 | | | | | | |
| 7 | 0.0 | 0.0 | .04 | 0.0 | .25 | .25 | 1.0 | | | | | |
| 8 | 0.0 | 0.0 | 0.0 | 0.0 | .25 | .25 | .25 | 1.0 | | | | |
| 9 | .10 | 0.0 | 0.0 | 0.0 | .10 | 0.0 | 0.0 | 0.0 | 1.0 | | | |
| 10 | 0.0 | 0.0 | .10 | 0.0 | 0.0 | 0.0 | .10 | 0.0 | 0.0 | 1.0 | | |
| 11 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | |
| 12 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |

Appendix B:
Sample Size = 120, Number of Iterations per PA = 500

99th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 7 | .7 |
| 1 | 57 | 5.7 |
| 2 | 763 | 76.3 |
| 3 | 150 | 15.0 |
| 4 | 18 | 1.8 |
| 5 | 5 | .5 |

95th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 1 | .1 |
| 1 | 23 | 2.3 |
| 2 | 642 | 64.2 |
| 3 | 247 | 24.7 |
| 4 | 66 | 6.6 |
| 5 | 18 | 1.8 |
| 6 | 3 | .3 |

90th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 14 | 1.4 |
| 2 | 533 | 53.3 |
| 3 | 306 | 30.6 |
| 4 | 103 | 10.3 |
| 5 | 37 | 3.7 |
| 6 | 6 | .6 |
| 8 | 1 | .1 |

50th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 194 | 19.4 |
| 3 | 262 | 26.2 |
| 4 | 243 | 24.3 |
| 5 | 166 | 16.6 |
| 6 | 80 | 8.0 |
| 7 | 52 | 5.2 |
| 8 | 3 | .3 |

99th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 55 | 5.5 |
| 1 | 127 | 12.7 |
| 2 | 740 | 74.0 |
| 3 | 74 | 7.4 |
| 4 | 4 | .4 |

95th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 11 | 1.1 |
| 1 | 64 | 6.4 |
| 2 | 753 | 75.3 |
| 3 | 148 | 14.8 |
| 4 | 20 | 2.0 |
| 5 | 4 | .4 |

90th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 5 | .5 |
| 1 | 33 | 3.3 |
| 2 | 716 | 71.6 |
| 3 | 192 | 19.2 |
| 4 | 41 | 4.1 |
| 5 | 11 | 1.1 |
| 6 | 2 | .2 |

50th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 4 | .4 |
| 2 | 344 | 34.4 |
| 3 | 297 | 29.7 |
| 4 | 191 | 19.1 |
| 5 | 99 | 9.9 |
| 6 | 37 | 3.7 |
| 7 | 25 | 2.5 |
| 8 | 3 | .3 |

Appendix C:
Sample Size = 120, Number of Iterations per PA = 250

99th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 5 | .5 |
| 1 | 50 | 5.0 |
| 2 | 757 | 75.7 |
| 3 | 167 | 16.7 |
| 4 | 19 | 1.9 |
| 5 | 1 | .1 |
| 6 | 1 | .1 |

95th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 1 | .1 |
| 1 | 21 | 2.1 |
| 2 | 587 | 58.7 |
| 3 | 302 | 30.2 |
| 4 | 76 | 7.6 |
| 5 | 9 | .9 |
| 6 | 4 | .4 |

90th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 9 | .9 |
| 2 | 502 | 50.2 |
| 3 | 327 | 32.7 |
| 4 | 126 | 12.6 |
| 5 | 24 | 2.4 |
| 6 | 8 | .8 |
| 7 | 4 | .4 |

50th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 190 | 19.0 |
| 3 | 260 | 26.0 |
| 4 | 251 | 25.1 |
| 5 | 149 | 14.9 |
| 6 | 89 | 8.9 |
| 7 | 59 | 5.9 |
| 8 | 2 | .2 |

99th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 66 | 6.6 |
| 1 | 123 | 12.3 |
| 2 | 732 | 73.2 |
| 3 | 72 | 7.2 |
| 4 | 7 | .7 |

95th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 4 | .4 |
| 1 | 56 | 5.6 |
| 2 | 759 | 75.9 |
| 3 | 158 | 15.8 |
| 4 | 21 | 2.1 |
| 5 | 1 | .1 |
| 6 | 1 | .1 |

90th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 0 | 1 | .1 |
| 1 | 33 | 3.3 |
| 2 | 691 | 69.1 |
| 3 | 226 | 22.6 |
| 4 | 44 | 4.4 |
| 5 | 4 | .4 |
| 6 | 1 | .1 |

50th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 3 | .3 |
| 2 | 331 | 33.1 |
| 3 | 313 | 31.3 |
| 4 | 182 | 18.2 |
| 5 | 95 | 9.5 |
| 6 | 47 | 4.7 |
| 7 | 27 | 2.7 |
| 8 | 2 | .2 |

Appendix D:
Sample Size = 240, Number of Iterations per PA = 500

99th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 667 | 66.7 |
| 3 | 281 | 28.1 |
| 4 | 47 | 4.7 |
| 5 | 4 | .4 |

95th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 503 | 50.3 |
| 3 | 347 | 34.7 |
| 4 | 125 | 12.5 |
| 5 | 22 | 2.2 |
| 6 | 2 | .2 |

90th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 413 | 41.3 |
| 3 | 370 | 37.0 |
| 4 | 169 | 16.9 |
| 5 | 41 | 4.1 |
| 6 | 6 | .6 |

50th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 121 | 12.1 |
| 3 | 318 | 31.8 |
| 4 | 274 | 27.4 |
| 5 | 190 | 19.0 |
| 6 | 80 | 8.0 |
| 7 | 17 | 1.7 |

99th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 831 | 83.1 |
| 3 | 150 | 15.0 |
| 4 | 18 | 1.8 |

95th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 674 | 67.4 |
| 3 | 275 | 27.5 |
| 4 | 45 | 4.5 |
| 5 | 5 | .5 |

90th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 597 | 59.7 |
| 3 | 301 | 30.1 |
| 4 | 84 | 8.4 |
| 5 | 16 | 1.6 |
| 6 | 1 | .1 |

50th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 266 | 26.6 |
| 3 | 352 | 35.2 |
| 4 | 208 | 20.8 |
| 5 | 117 | 11.7 |
| 6 | 49 | 4.9 |
| 7 | 8 | .8 |

Appendix E:
Sample Size = 240, Number of Iterations per PA = 250

99th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 704 | 70.4 |
| 3 | 243 | 24.3 |
| 4 | 45 | 4.5 |
| 5 | 7 | .7 |

95th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 522 | 52.2 |
| 3 | 346 | 34.6 |
| 4 | 117 | 11.7 |
| 5 | 15 | 1.5 |

90th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 420 | 42.0 |
| 3 | 377 | 37.7 |
| 4 | 166 | 16.6 |
| 5 | 33 | 3.3 |
| 6 | 4 | .4 |

50th Percentile, Any Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 134 | 13.4 |
| 3 | 280 | 28.0 |
| 4 | 296 | 29.6 |
| 5 | 190 | 19.0 |
| 6 | 83 | 8.3 |
| 7 | 17 | 1.7 |

99th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 857 | 85.7 |
| 3 | 128 | 12.8 |
| 4 | 11 | 1.1 |
| 5 | 3 | .3 |

95th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 712 | 71.2 |
| 3 | 231 | 23.1 |
| 4 | 48 | 4.8 |
| 5 | 8 | .8 |

90th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 1 | 1 | .1 |
| 2 | 623 | 62.3 |
| 3 | 281 | 28.1 |
| 4 | 83 | 8.3 |
| 5 | 12 | 1.2 |

50th Percentile, 10% Difference

| Factors | Frequency | Percent |
|---|---|---|
| 2 | 261 | 26.1 |
| 3 | 317 | 31.7 |
| 4 | 261 | 26.1 |
| 5 | 107 | 10.7 |
| 6 | 46 | 4.6 |
| 7 | 8 | .8 |