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Mathematical Models of *Zea mays*: Grain Yield and Aboveground Biomass Applied to Ear Flex and within Row Spacing Variability

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MATHEMATICAL MODELS OF *Zea mays*: GRAIN YIELD AND ABOVEGROUND BIOMASS APPLIED TO EAR FLEX AND WITHIN ROW SPACING VARIABILITY

A Thesis
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Master of Science

By
Todd Curtis Ballard
December 2008

MATHEMATICAL MODELS OF *Zea mays*: GRAIN YIELD AND ABOVEGROUND BIOMASS APPLIED TO EAR FLEX AND WITHIN ROW SPACING VARIABILITY

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MATHEMATICAL MODELS OF *Zea mays*: GRAIN YIELD AND ABOVE GROUND BIOMASS APPLIED TO EAR FLEX AND WITHIN ROW SPACING VARIABILITY

Todd Curtis Ballard December 2008

Directed by: Dr. William T. Willian, Dr. Elmer Gray, Dr. Martin Stone, and Dr. Marvin W. Russell

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Field studies were conducted during the summers of 2007 and 2008 at the Agricultural Research and Education Complex, Western Kentucky University, Warren County, KY and commercial production fields in Caldwell County, KY, Warrick County IN, and Vanderburgh County, IN. The goals of these studies were to further validate the Duncan grain yield model, the Russell aboveground biomass model, and to study the effect of inconsistent spacing within rows on *Zea mays* L. yield. Plant spacing other than uniform decreases grain yield and profitability. The population experiments conducted at the Warren County location were a randomized complete block design with three planting densities, three varieties (c.v. DeKalb DKC6547, DeKalb DKC6346, DeKalb DKC6478) in 2007 and (DeKalb DKC6478, DeKalb DKC6342, and DeKalb DKC6544) in 2008, and three replications. Seeds were planted in rows 76 cm apart and 9.1 m long with four rows per plot in a no-till system on a Crider Silt Loam with pH of 6.8 and 1.5% organic matter. The effect of variable within row spacing was evaluated in commercial production fields by randomly selecting five adjacent rows of 5.3 meters in length at each

location. Grain yield for each row was then curve fitted both linearly and exponentially.

Minimizing interspecies competition was essential to evaluating the effects of competition within *Zea mays* L. A burn-down application of 2,4-D and glyphosate was used prior to planting. The most common weeds in the plots were *Sorghum halepense* L. (johnsongrass)*, Trifolium repens* L. (white clover)*,* and *Taraxacum officinale* L. (common dandelion) . Glyphosate was reapplied throughout the growing season due to reemergence of *S. halepense* and *Ipomoea hederacea* Jacq. (ivyleaf morningglory).

The weight of each ear was recorded and one row from each plot was randomly selected to shell. The moisture content was measured from a subsample twice each row using an electrical conductivity moisture meter. The mean of the two moisture readings was used as the moisture content from the plot. Cob weights from shelled ears were recorded to determine the grain/cob mass ratio. This ratio was used to project the grain weight for the remaining harvested rows.

Duncan's grain yield model and Russell's biomass model were curve fitted to the data for areas of 0.00040 hectares at the $p < 0.05$ significance level or greater in all population density plots. Individual plant grain masses were curve fitted to Duncan's model with $p < 0.05$ significance in 3 out of 15 plots. Grain mass was negatively correlated ($R < 0$) with standard deviation of within row spacing in 14 of 15 plots. A linear fit to this trend was significant in only 2 of 15 plots. The Duncan yield curve and the Russell aboveground biomass model fit all 6 genotype by environment interactions for 2007 and 2008 to the $\alpha = 0.05$ level of confidence when evaluated over a 5.3 meter length on 76.2 cm wide rows. Individual plants fit linearly at $\alpha = 0.05$ in 9 out of 15 plots. Individual plants fit the Duncan yield curve at $\alpha = 0.05$ in 4 out of 15 plots.

Standard deviation of within row spacing fit grain yield loss significantly at $\alpha = 0.05$ in two of 15 plots. The individual plant spacing and local population density collectively fit nine plots significantly at $\alpha = 0.05$ or better.

CHAPTER 1

INTRODUCTION

Zea mays L. is an annual species of the Pocaceae family. The origin of corn is thought by many to be Southern Mexico or Northern Guatemala (Smith 2004). Likely progenitors are *Z. diploperennis* Iltis*, Z. perennis* Hitchcock*, Z. luxurians* (Durieu and Ascherson) Bird*, and Z. mays* L. ssp. *huetenansensis, Z. mays* L. ssp. *mexicana, and Z. mays* L. ssp. *parviglumis*. Collectively these species are commonly known as teosinte(s) (Smith 2004, Doebley 2008).

Dent corn is a major agronomic crop in much of the world. In the United States production is most prominent in Eastern NE, IA, Southern MN, IL, IN, and Southern MI (National Agricultural Statistics Service 1992). Corn production in Kentucky is concentrated in the western third of the state (NASS, 1992).

A cost efficient seeding rate has gained importance in the last ten years due to their increased seed costs. The most prominent being fees for genetically engineered varieties. Excessive plant density or poorly spaced plants result in neighboring corn plants unduly competing. Seeding densities that are below optimum rates result in inefficient land use and a thin canopy, which allows unimpeded rainfall to impact the soil, detaching surface particles, and resulting in erosion. A thin leaf canopy also allows an increased rate of evaporation from the soil and more light to reach seedling weeds.

The objectives of this research were:

(a) To determine optimum planting densities for dent corn varieties under the conditions at the WKU Agricultural Research and Education Center

(b) To further validate the Duncan grain yield model

(c) To review the use of Shinozaki and Kira's (1956) aboveground biomass model for certain other agronomic plants for use in corn. The same equation was covered independently by M.W. Russell during curve fitting studies of corn biomass data in the literature (from this point forward applying the Shinozaki and Kira biomass model to corn will be referred to as the Russell biomass model (Russell 1979) as demonstrated in literature review)

(d) To combine Duncan's grain yield model and Russell's aboveground biomass model for aboveground harvest index and stover yield.

CHAPTER II

LITERATURE REVIEW

An early model of aboveground biomass production is that of the Mitsherlich equation $M = M_\infty(1 - e^{-kX})$ (Overman and Scholtz, 2007). In production experiments over the last 100 years, X the independent variable, denoted many different variables related to soil fertility. M_∞ was the asymptotic maximum production. M is the predicted total biomass production per unit area at a given density.

Duncan (1958) saw a need to develop a model for grain production in corn for a given set of environmental and genetic circumstances. He found the exponential decay model, $y = ae^{-bX}$, to be appropriate. This function is found in many scientific fields, including Bier's law in chemistry, radioactive decay in atomic physics, and uninhibited population growth in ecology (Russell, personal communication). For the purposes of Duncan's application of this equation to the corn plant, a and b are parameters defined by the genotype by environment interaction (Duncan, 1958). In this paper the independent variable will always be P, plants per unit area (hectare). The dependent variable, y, is the expected grain yield per plant. By multiplying y by P a correlated and more useful model is obtained $Y = aPe^{-bP}$. This model will give the average production per unit area at a given population. The population which produces maximum yield occurs when $\frac{dY}{dP} = 0$. Therefore the population which produces maximum grain yield is $\frac{1}{b}$ (Newton, 1686 and Duncan, 1984).

Duncan (Russell, personal communication) noted that the Mitsherlich equation was not appropriate for use with corn. Also “This model was shown to apply to warm-season perennial grass for harvest intervals up to about 6 weeks, but failed for longer growth intervals and did not apply to annual grasses such as corn” (Overman and Scholtz, 2002). Duncan’s observation inspired Russell to develop a correct total aboveground biomass model for corn plant density.

The Russell above ground biomass model for corn is the same as the tuber yield model of Shinozaki and Kira: $\frac{1}{m} = \mathrm{BP} + \mathrm{A}$ (Russell, 1979), B and A are parameters defined by genotype and environment, P is the population density in plants per unit area and m is the expected aboveground biomass for a single plant. The above ground biomass, M, produced per unit area is obtained by multiplying m and P. Therefore, $\mathrm{M} = \mathrm{P}/(\mathrm{BP} + \mathrm{A})$.

With both a grain yield per population model and an above ground biomass per population model the aboveground biomass can be obtained by the equation

$$\mathrm{H} = \frac{Y}{M} = \mathrm{a}e^{-\mathrm{bP}}(\mathrm{BP}+\mathrm{A}).$$

Grain yield losses have been noted due to within row spacing variability (Nielson 2001). He used a linear model for losses where the statistic $\sigma \geq 5.1$ cm. Achieving $\sigma < 5.1$ cm may not be possible with an approximate 95% germination and currently available mechanical planters.

Using Duncan’s grain yield model the population density which produces maximum grain yield can be identified, but it does not identify the population density which produces the maximum economic yield for grain production.

Duncan stated “a linear relationship exists between the logarithm of the average plant yield and the population.” The strength of this relationship was demonstrated in a table of r values from experiments replicated at least four times in NE, OH, IL, and IN. These values range from 0.9885 to 0.9991 (Duncan, 1958). The only published data Duncan found which did not fit to a 0.05 level of significance was a study of within row competition. He found variation in data from southern states tended to be greater than that from the Corn Belt states listed (Duncan, 1958).

Two challenges exist in applying Duncan’s (1958) work to contemporary production. First, the range of relevance was stated to be between 12,000 and 62,000 plants ha^{-1}. Since most fields in humid climates are planted above 62,000 plants ha^{-1} (Klein and Lyon, 1997 and Russell, personal communication), higher population densities should be investigated. It was important to note that in the hypothetical example given by Duncan (1958) that the population which produced maximum yield is only 27,000 plants ha^{-1}. Under appropriate fertilization and water availability today’s hybrids maximize grain yield around 74,000 plants ha^{-1}(Klein and Lyon, 1997). Sixty two thousand plants ha^{-1} is more than twice the peak population in the hypothetical example. The second challenge is a matter of making the mathematics more cumbersome than necessary. Duncan (1958) uses a log base 10 rather than the natural log. This approach required a conversion when finding the P_{max}. When using a ln the P_{max} is $\frac{1}{b}$ (Duncan, 1984).

Duncan mentioned the influence of nitrogen, phosphorus, and potassium’s on the ln y data results. An increase in N decreased of the slope of the log of yield per plant. He stated that studies on P and K levels on the population which produces maximum yield would be appropriate as no data were found on their effects to the curve (Duncan, 1958).

Duncan (1984) investigated the effects of crowding on individual plants. Which resulted in the grain yield model as demonstrated in his earlier article. Between publications of the Duncan articles he and others continued to use the model. Only in the case of sweet corn did the model fail to fit with an r value of 0.98 or greater. However, plant population density was not a complete descriptor of intraspecies plant competition. Distance between rows as well as the variation in within row spacing also influence overall interference of the individual plants (Duncan, 1984).

Three terms were introduced by Duncan (1984) to discuss the crowding effect. C is a numerical value for crowding. When two plants are “in contact” the C value for one of the plants on the other is 1. Duncan (1984) defines in contact as zero separation. When two plants are far enough apart for their competition to be negligible the distance apart is known as DMAX. The effect on yield due to C is known as E. From these terms Duncan explains the model in terms of e: $y = y_0e^{EC}$. The interference effect terms are displayed in Table 1.

Two more variables were introduced to find the C value for any given competing plant (Duncan, 1984). The rate of change in the value of C is known as α. This rate is determined by the distance DMAX. SF is the separation factor: SF = (DMAX – separation)/DMAX. Therefore, the overall value of C is ∑(SF^α) from plant 1 to n. Duncan suggests using three meters as the value of DMAX. He concluded that the most efficient use of space for a planting pattern for plants surrounding a single corn plant is that of a hexagon. This conclusion lends itself to narrow rows, planting each row offset to the next by 50% of the distance between plants.

Work was continued on the Duncan grain yield model by Carmer and Jackobs (1965). They acknowledged a true value of the Duncan grain yield model lies in evaluating fertilizer treatments. By using the model for variety trials and fertilizer experiments they found that varieties "could then be compared on the basis of their highest yielding or optimum plant densities rather than at some arbitrarily selected density which favored some and handicapped others." Linear transformation of the exponential equation skewed the best fit in values of e or less. Due to transformation, an estimate of experimental error was needed. An important note made by Carmer and Jackobs (1965) is that planting at the true optimal density for a given season is "difficult or impossible". The percentage of the optimal yield can be found using

$\frac{Y_P}{Y_{\max}} = 100(\mathrm{Pe}\wedge^{1\text{-}P})$. Seven out of eight hybrids tested fit the Duncan grain yield at 0.05 significance (Carmer and Jackobs, 1965). The eighth fit the model at 0.05 significance at 3 out of 4 of the populations tested. They failed to mention if the problematic density was the lowest, highest or somewhere in between.

The simulated yield at any population density contains some experimental error. To estimate the precision of the P_m a, and b values fitted to the data, the inequality: $\delta = |N^{\wedge}\text{-}N| < \gamma\, P/100$ can be used, where N is the true value of P or the parameters. γ is the goal percentage value to be within in the estimate and δ can be standardized so that a z table can be used (Carmer, 1970). Through several table demonstrations Carmer concluded a and P_m are estimated with less precision than b and Y_m. A choice of four

Table 1. Model terms used to describe the effect of one corn plants' interference on another[1].

C	Crowding effect of 1 plant to another
DMAX	Distance required between plants for C to be negligible
E	Effect on yield due to C
y_0	Individual plants yield when no interference occurs
SF	(DMAX-distance between plants)/DMAX
α	Rate of change to $\frac{dC}{dSF}$

[1] Duncan (1984).

densities results in the greatest precision of P_m.

Utilizing the values of experimental error, Carmer (1970) evaluated three methods of regression, a non-linear method used by Carmer and Jackobs (1965), the linear transformation, and a second iterative least squares fit method of the exponential function. To evaluate these methods 9,000 experiments were computer simulated using nine experimental designs, five magnitudes of error, and two error structures. They were separated into groups of 100 replicated experiments for each of the 90 combinations. The true fit model was recorded for each. Prior to the data analysis it was determined that the per unit area yield variance would be used rather than the per plant variance. This implies a multiple plant average variance was used because researchers rarely record individual plant yields (Carmer, 1970).

Twenty seven thousand data sets resulted from the simulated experiments due to the different densities for each. Computational method "one" was used to find the least squares regression of a linear transformation. Computational method "two" was non-linear regression using the $y = ae^{-bP}$ form of the model. Method "three" was non-linear regression using the $Y = Pae^{-bP}$ form of the model. No preference for the quality of fit was found by Carmer (1970) for any of the methods. This implies that the computational method having the most accessible software is the most appropriate to use.

A competition-density effect was proposed (Shinozaki and Kira, 1956) for duckweed (*Lemna minor* L.). Duckweed fit hyperbolic growth at 0.05 significance level. Shinozaki and Kira (1956) studied intraspecific competition from the perspective of total aboveground biomass. Their aim was to study the same competition density effect and its mathematical derivative, the yield-density effect applied to higher plants ($\frac{dC}{dD}$ = yield density effect). The yield density effect was compared to Mitsherlich's law. In order to satisfy a study of population density as it relates to intraspecific competition, the following must be true; seeds are planted simultaneously, fertilizers and water are supplied amply or equally to all plots, and harvest must occur simultaneously.

Shinozaki and Kira (1956) harvested aboveground biomass which was sampled multiple times in the vegetative growth stage of several species. At harvest both aboveground biomass and reproductive masses were gathered. All equations tested fit the curves in Table 2. They evaluated the overall aboveground biomass model on several species. The R^2 values were 0.95 or greater for *Brassica rapa* L. (turnip), *Glycine max* L. (soybeans), *Vigna angularis* Ohwi and Ohashi (azuki beans), *Daucus carota* L. (carrots), *Trifolium subterranean* L. (subterranean clover), and *Pinus densiflora* Cheus and Chu (Japanese red pine) (Shinozaki and Kira, 1956). Russell (1979) agreed with Shinozaki and Kira upon finding corn aboveground biomass fitted the competition density effect. Shinozaki and Kira (1956) were also able to significantly fit ($\alpha = 0.05$) the aforementioned species to the sigmoid growth curve.

Comparison of the competition density yield curve and the Mitsherlich Law in two plots of *Sinapis alba* L. (white mustard) reported by Mitsherlich revealed the Shinozaki and Kira model fit with a higher R value (0.99) (Shinozaki and Kira, 1956). They found the Mitsherlich equation to be empirical and their competition density equation consistently fit biomass data better.

Unequal spacing variability can be caused by soil compaction, poorly maintained planters, water or pests moving seeds after planting, failed germination, incorrect planting plates, or excessive planter speed (Nielson, 2001). Excessive speeds result in higher population densities (Nielson, 2002).Excessive plant densities resulted in higher seed cost and lost yield if the germinated population exceeds P_{max} of the Duncan grain yield model.

The standard deviation of the distances between plants is the best measure of plant spacing variability. Sixteen percent of 350 fields he studied had a PSV (Plant Spacing Variability) of 7 cm or less, 60% had a PSV of 10 cm – 12 cm and 24% of the fields had a PSV > 12 cm. Considering the worst case observed scenario of 10 kg of grain yield loss cm^{-1} of standard deviation, large economic losses occurred when PSV > 7 cm (Nielson, 2001) .

Popp et al. (2006) considered economic aspects of corn plant population density. Their research objective was to determine whether planting short-season maize in late March or early April created an economic gain compared to later planting. Arkansas producers have been planting short season maize at later dates to allow time to work on their other crop production. However, later plantings were injured by the July drought. Short season hybrids planted in March and April reduced irrigation costs.

Popp et al. (2006) used the Mitscherlich equation to predict corn yield. The profit equation: profit = Y(Price) – cost (plant population density) was used. They used average season price for the Fayetteville, Arkansas area. Eighty four and 109 day relative maturity (RM) hybrids planted early were the most profitable. The 84 day RM hybrids outperformed the 109 day hybrids in grain yield, but the seed cost of the 84 day hybrids reduced their profitability.

Popp et al. (2006) also performed an analysis of the financial risk associated with producing non-irrigated corn in Arkansas and Louisiana. “When starting, available

Table 2. Mathematical relationships for aboveground biomass, reproductive yield, and aboveground biomass growth in multiple plant species.*

Effect	Equation	Description
Competition Density	$M = \frac{P}{BP + A}$	Aboveground biomass per unit area as affected by population density in plants with harvest intervals greater than six weeks.
Yield Density	$Y = aPe^{-bP}$	Reproductive end of season mass as affected by population density.
Sigmoid Growth	$m_t = \frac{m_f}{1 + ke^{-\lambda t}}$	Aboveground biomass at a given time t m_f = harvest biomass

*Observed by Shinozaki and Kira (1956)

water was 7.9 cm, average gross margins were less than \$15.07 ha^{-1} ,and the risk of financial loss exceeded 40%." When such a significant likelihood of financial loss exists, without a chance of significant reward, another crop, such as *Sorghum bicolor* L. (grain sorghum) should be considered.

Klein and Lyon (1997) developed a planting density guide for Nebraska producers. Water availability as well as accumulated growing degree days varied greatly from the eastern to the western side of the state leading to a wide range of appropriate planting densities. The wide range of appropriate planting densities implied a wide range of P_{max}.

There are more factors leading to the amount of water in a field than just the transpiration from the leaves. These include runoff, soil evaporation, and weeds (Klein and Lyon 1997). Available water in the soil profile prior to planting must be taken into consideration when contemplating planting rate. The researchers found that yields in Nebraska were maximized at 20,000, 30,000, and 40,000 established plants per hectare for starting available water levels of 7.9, 16, and 24 cm; respectively" (Klein and Lyon 1997).

Duncan's grain yield model and Russell's biomass model were evaluated using data generated prior to their discovery. In data from 1919 (Montgomery) Duncan's and Russell's model fit with an r^2 of 0.99 and 0.95 respectively. These r values demonstrated that their models fit open-pollinated varieties.

Corn ear flex describes the genetic plasticity of ear development in *Zea mays* L. Flex can describe the plasticity of ear length, ear girth, and number of ears on a plant (Thomison, 1990). High, medium, and low flex indicate the capacity to adapt to an environment. Hence, high flex cultivars are more likely to produce grain at minimal competition when compared to medium flex. Similarly, medium flex cultivars are adaptable to a wider range of environments than low flex cultivars (Thomison, 1990). Flex has not been shown to influence P_{max}.

CHAPTER III

MATERIALS AND METHODS

Population Density Experiment

A randomized complete block design was utilized with 9 treatments and 3 replications for the population density experiment. Treatments were: low flex (DeKalb cv. DKC6547 in 2007 and DeKalb cv. DKC6544 in 2008), medium flex (DeKalb cv. DKC6346 in 2007 and cv. DKC6342 in 2008), and high flex (DeKalb cv. DKC6478 in 2007 and DeKalb cv. DKC 6478 in 2008) flex corn cultivars. Varieties of the same flex were as close as could be provided in parentage by DeKalb (Monsanto). The parents could not be named by DeKalb due to their proprietary nature. Each cultivar was seeded at three populations: 29,728 seeds ha^{-1}, 60,886 seeds ha^{-1}, and 91,330 seeds ha^{-1}. Plot dimensions were 9.1 m long and had 4 rows of corn 76 cm apart.

The experiment was conducted at the Western Kentucky University Agricultural Research and Education Center (37^0 N and 86^0 W). The soil type was a Crider silt loam, 2-6% slope (fine silty mixed-active typic Paleudalf). Soil core samples were taken in April of 2007 and 2008. Soil tests results and recommendations from A & L Analytical Laboratories are shown in Appendix 2 (2007) and Appendix 3 (2008).

Based upon soil analysis, nitrogen (N) was applied at 224 kg ha^{-1} in 2007 using NH_4NO_3. Phosphorus (P_2O_5) was applied at 62 kg ha^{-1}. Potassium (K_2O) was applied at 45 kg ha^{-1}. Sulfur (S) was applied using elemental S at a rate of 11 kg ha^{-1}. In 2008 P was applied at a rate of 34 kg ha^{-1}. All other fertilization rates remained the same in 2008 as in 2007.

A burndown application of 2,4-D (0.6 kg a.i. ha^{-1}) and glyphosate (1.1 kg a.i. ha^{-1}) was applied both years. Weeds were controlled throughout the growing season with glyphosate applications. In-row spacing and plants per plot were recorded three weeks after emergence in 2007 and at harvest in 2008.

All plants within a 0.00040 ha area were harvested individually for grain in both 2007 and 2008, ears were placed in individual bags. The remaining above ground biomass was harvested in 0.00081 ha areas. Ear weights were recorded for individual plants. Half of the harvested ears were shelled to obtain a grain weight to cob weight ratio for each plot. Grain weights of the remaining intact ears were estimated from this ratio. Two moisture tests were completed per plot. Weight of grain was adjusted to 15% moisture. Similarly the overall above ground biomass weights were converted to 15% moisture after the remaining aboveground biomass was kiln dried for 48 hours to obtain minimal moisture levels.

Spacing Study

Fourteen out of 15 plots in the spacing study were subsampled from commercial fields. One plot was sampled from the medium flex population study in 2008. In 2007, four plots were located on the WKU AREC (cvs. Garst 84-52, Garst 84-97, and Garst 83-53), two in Caldwell County, KY near Dawson Springs (cv. Pioneeer 33M54), one near Elberfeld, IN (cv. Pioneer 33M55), and one North of the Evansville, IN airport (cv. Pioneer 33M55). In 2008, five plots were located on the WKU AREC (cvs. Crows 5176S-RR, Syngenta NK N72-Q6, and DeKalb DKC6342), one in Warrick County IN near Chandler (cv. Trisler T-7N53), and one on the same farm Caldwell County KY farm as in year 2007 (cv. Pioneer 33M54). The plots were selected randomly within each field of interest by the producer. They were all between 10 m and 40 m from any border rows to eliminate the border effect. Fourteen of the 15 fields were planted at 74,000 seeds ha^{-1}. In 2008, WKU plot 5 was planted at 61,000 seeds ha^{-1} with DeKalb DKC6342.

Five adjacent rows 5.3 m long were harvested in a similar fashion to the population density study. Individual ear(s) were placed in an individual bag with the location, variety row number, and plant number within the row labeled on the bag. Distances between plants and the number of plants within the 5.3 m length were recorded on the harvest date. One row from each plot was shelled to obtain the grain to cob weight ratio. This ratio was used to project the grain weight of the remaining rows in the plot. The moisture readings were obtained from ears not in the row used to find the grain/cob ratio. All weights were then standardized to 15% moisture.

Population Density

In the plant population study all grain data per 0.00040 ha^{-1} were curve fit to the Duncan yield curve and tested at 0.05 and 0.01 α significance levels. All varieties had 18 observations (two rows per plot). Aboveground biomass data per .00081 ha was curve fit to the Russell aboveground biomass model and tested at the 0.05 and 0.01 α significance levels. All varieties had nine observations (one per plot).

Spacing Study

Data from the spacing study were curve fit linearly with the standard deviation of within row spacing, linearly to individual plant spacing, and to the Duncan yield curve using the population density imposed by the two closest neighboring plants. All data analysis was completed using Microsoft Excel 2003.

CHAPTER IV

RESULTS AND DISCUSSION

Population density

In 2007 and 2008, R values for the Duncan yield curve were significant at the 0.01 level for all cultivars. The R values (0.000040 ha) were -0.94 for low flex, -0.87 for medium flex, and -0.94 for high flex cultivars in 2007 and -0.98 for low flex -0.96 for medium flex and -0.98 for high flex cultivars, respectively in 2008 (Table 3). The R values of the individual plants observed within these areas were between -0.4 and -0.6. In the plant by plant observations more variability was evidenced than in the observations per unit area. The large number of degrees of freedom (350 for each cultivar) allowed the R values to remain significant at the 0.01 level.

The resulting equations per plant were (2007) $y = 240e^{-0.0221P}$ for the high flex cultivar, $y = 167e^{-.0165P}$ for the medium flex cultivar, and $y = 180e^{-0.018P}$ for the low flex cultivar in 2007 (Figure 1). In 2008 the equations were $y = 540e^{-0.0165P}$ for the high flex cultivar, $y = 563e^{-0.0178P}$ for the medium flex cultivar, and $y = 599e^{-0.0172P}$ for the low flex cultivar (Figure 2).

The $\frac{1}{b}$ value (population for maximum yield) for the high flex cultivar was 45,200 plants ha^{-1} in 2007. This would give a maximum Y value of 4020 kg ha^{-1}.

Figure 1. **Grain production of low, medium, and high flex cultivars of corn as influenced by plant population density in 2007.**

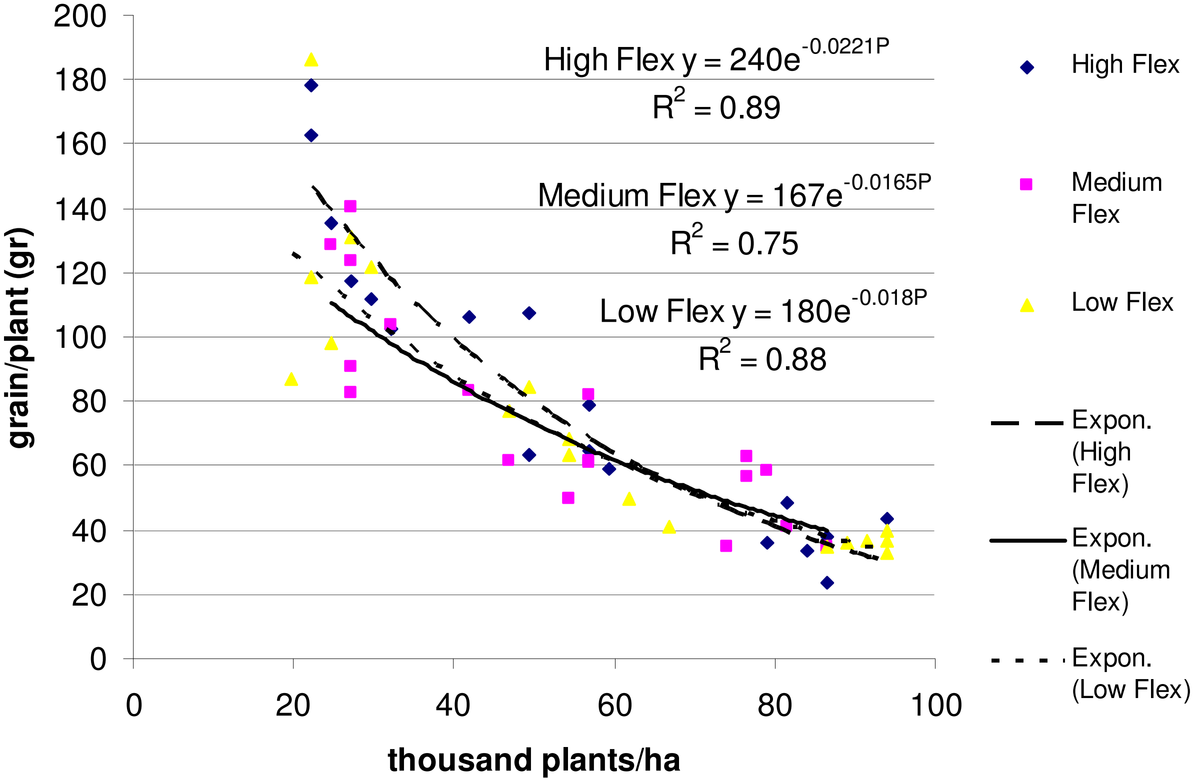


Figure 2. Grain production of low, medium, and high flex cultivars of corn as influenced by plant population density in 2008.

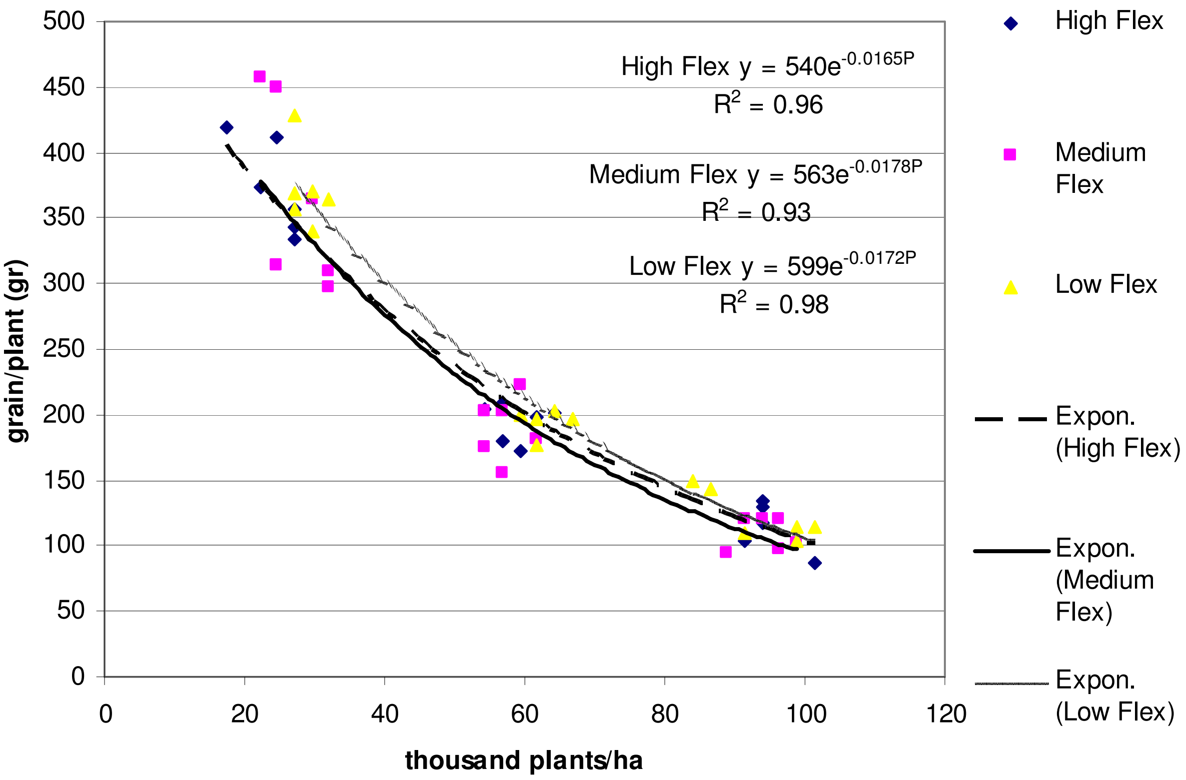


Table 3. Duncan yield equations for population density study of corn in 2007-2008.

Flex Trait and Year	a	b	R	y	Y	P_{max}	Y_m
***High Flex 2007	240	0.0221	-0.94	$y = 240e^{-0.0221P}$	$Y = 240Pe^{.0221P}$	45,200 P/ha	4020 kg/ha
*Medium Flex 2007	167	0.0167	-0.87	$y = 167e^{-0.0167P}$	$Y = 167.04Pe^{.0165P}$	59,900 P/ha	3680 kg/ha
**Low Flex 2007	180	0.018	-0.94	$y = 180e^{-0.018P}$	$Y = 180Pe^{.018P}$	55,600 P/ha	3680 kg/ha
*High Flex 2008	540	0.0165	-0.98	$y = 540e^{-.0165P}$	$Y = 540Pe^{-.0165P}$	60,600 P/ha	12,000 kg/ha
***Medium Flex 2008	563	0.0178	-0.96	$y = 562e^{-0.0178P}$	$Y = 563Pe^{-0.0178P}$	56,200 P/ha	11,600 kg/ha
**Low Flex 2008	599	0.0172	-0.98	$y = 599e^{-0.0172P}$	$Y = 599Pe^{-0.0172P}$	58,100 P/ha	12,800 kg/ha

* Highest P_{max} for year

** Second highest P_{max} for year

*** Lowest P_{max} for year

Likewise for the medium flex cultivar $\frac{1}{b}$ was 59,900 in 2007. The maximum Y at that population would be 3680 kg ha^{-1}. The low flex cultivar would have maximized its Y under the 2007 conditions at a population of 55,600 plants ha^{-1}. The low flex maximum 2007 yield was 3680 kg ha^{-1}. Maximum yield population densities in 2008 were 58,100 plants ha^{-1} for low flex (12,800 kg ha^{-1}), 56,200 plants ha^{-1} for medium flex (Y = 11,600), and 60,600 plants ha^{-1} (Y = 12,000 kg ha^{-1}) for high flex.

In 2007 and 2008, the R values for the Russell biomass equation were also significant at the 0.01 level. In 2007, R values (0.000081 ha) were 0.98 for the low flex cultivar, 0.92 for the medium flex cultivar, and 0.94 for the high flex cultivar. In 2008 R values were 0.92 for the high flex cultivar, 0.92 for the medium flex cultivar, and 0.96 for the low flex cultivar.

Total biomass equations per plant were as follows. The high flex cultivar m equation is $m = \frac{1}{0.0001P - 0.00009}$ in 2007 and m = $\frac{1}{0.00007P + 0.00002}$ in 2008. The medium flex variety mass per plant is $m = \frac{1}{0.00009P + 0.0006}$ in 2007 and m = $\frac{1}{0.00008P - 0.00005}$ in 2008. The low flex variety aboveground biomass per plant is expressed as $m = \frac{1}{0.00009P + .00005}$ in 2007 and m = $\frac{1}{0.00007P - 0.00004}$ in 2008 (Table 4). Figures 3 and 4 display the linear form of the Russell biomass model $\frac{1}{m} = AP + B$ for 2007 and 2008 respectively.

Table 4. Russell biomass equation results for corn in 2007 and 2008.

Flex Trait and Year	A	B	m $(\frac{gr}{plant})$	$\frac{1}{m}$ $(\frac{plant}{gr})$	M $(\frac{gr}{ha})$
High Flex 2007	0.0001	-0.00009	$m = \frac{1}{0.0001P - 0.00009}$	$\frac{1}{m} = 0.0001P - 0.00009$	$M = \frac{P}{0.0001P - 0.00009}$
Medium Flex 2007	0.00009	0.0006	$m = \frac{1}{0.00009P + 0.0006}$	$\frac{1}{m} = 0.00009 + 0.0006$	$M = \frac{P}{0.00009P + 0.0006}$
Low Flex 2007	0.00009	0.0005	$m = \frac{1}{0.00009P + 0.0005}$	$\frac{1}{m} = 0.00009P + 0.0005$	$M = \frac{P}{0.00009P + 0.0005}$
High Flex 2008	0.00007	0.00002	$m = \frac{1}{0.00007P + 0.00002}$	$\frac{1}{m} = 0.00007P + 0.00002$	$M = \frac{P}{0.00007P + 0.0002}$
Medium Flex 2008	0.00008	-0.00005	$m = \frac{1}{0.00008P - 0.00005}$	$\frac{1}{m} = 0.00008P - 0.00005$	$M = \frac{P}{0.00008P - 0.00005}$
Low Flex 2008	0.00007	-0.00004	$m = \frac{1}{0.00007P - 0.00004}$	$\frac{1}{m} = 0.00007P - 0.00004$	$M = \frac{P}{0.00007P - 0.00004}$

Figure 3. Multiplicative inverse of aboveground biomass of high, medium, and low cultivars of corn as influenced by plant population density in 2007.

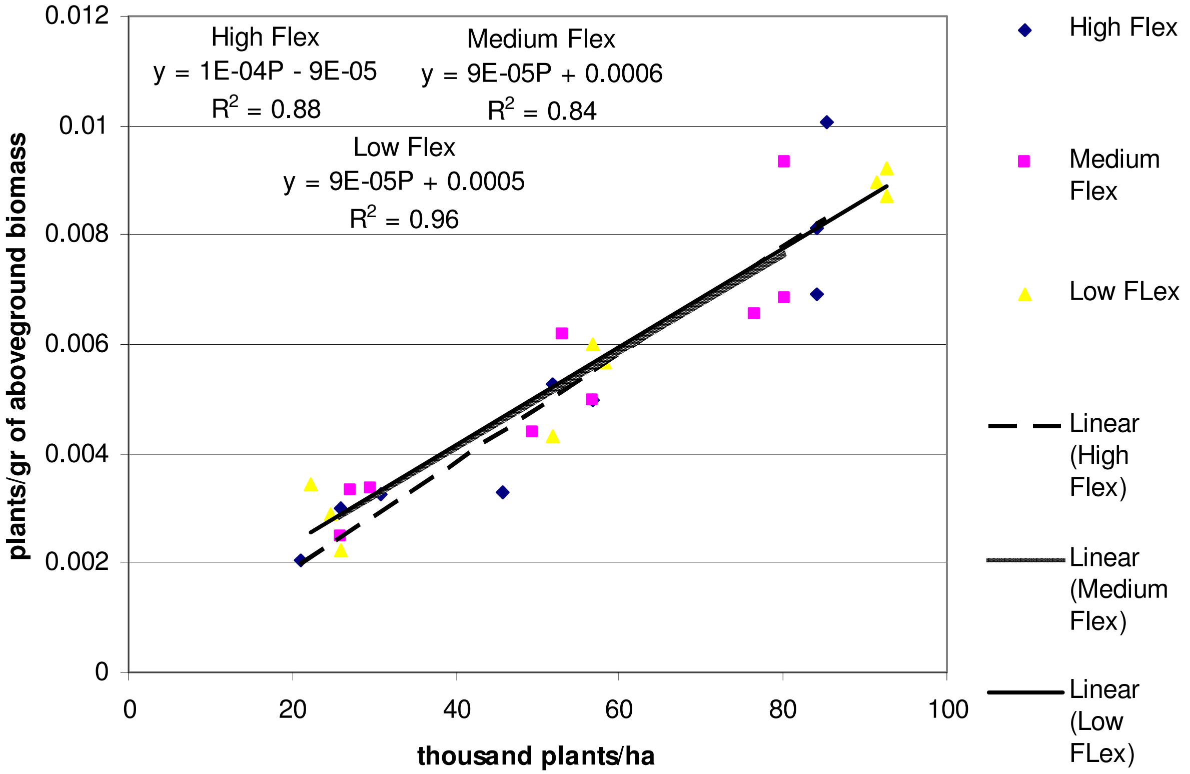
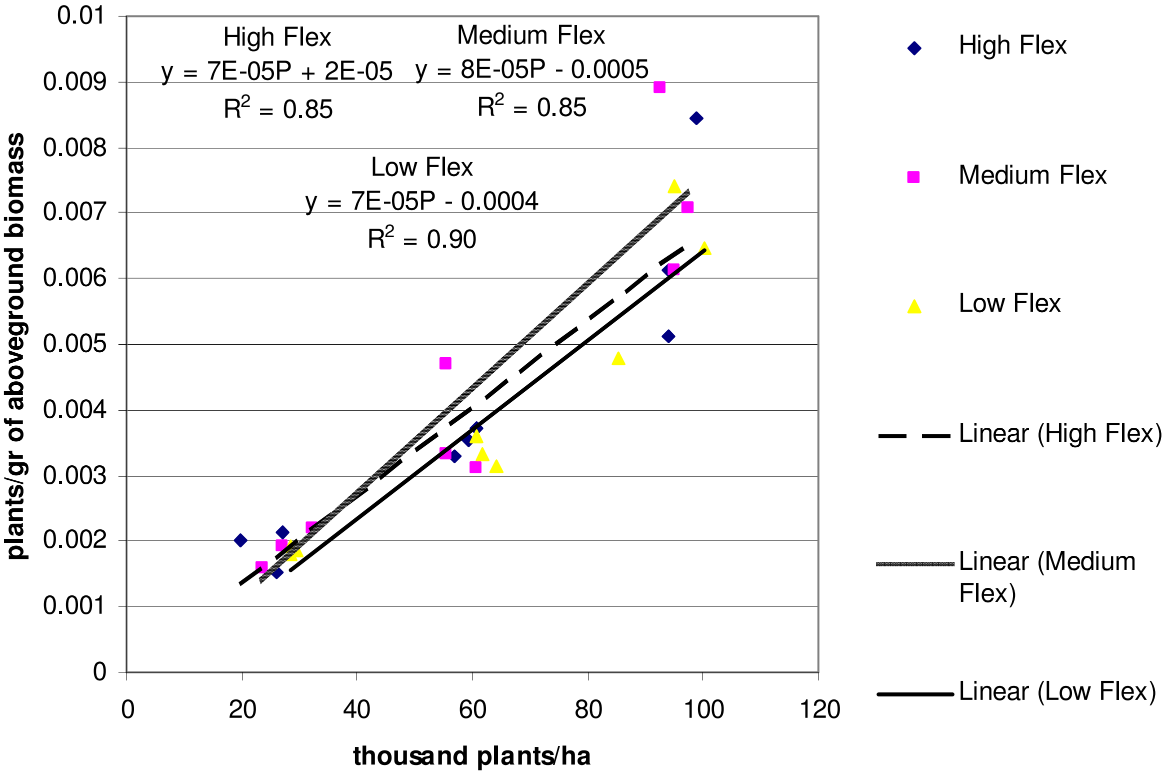


Figure 4. Multiplicative inverse of aboveground biomass of high flex, medium flex, and low flex cultivars of corn as influenced by plant population density in 2008.



Combining the Duncan and Russell Models

The models can be combined and utilized in several ways. The harvest index of any given population can be predicted using $\frac{y}{m}$ or $\frac{Y}{M}$, both result in $[ae^{-bP}](AP + B)$. The stover yield ha^{-1} is given as M – Y. Also economic implications arise from planting above or below the maximum grain yield. Net grain profit can be expressed as

$$N_P = -aPVe^{-bP} - \mu S - \Omega$$

where μ is the cost of 1000 seeds, S is the number of seeds needed to reach the desired population density at harvest, and Ω are the fixed costs associated with production at the given fertility level and V is the value of grain. Fixed costs include fertilizer, equipment depreciation and maintenance, fuel, and labor. The population which produces the maximum field net profit occurs when $\frac{dN}{dP} = 0$. Data in Appendix 1 demonstrate some observed relationships both within and between the models (Russell, personal communication). The Duncan and Russell models can be used to determine potential energy production utilizing grain and stover.

Standard Deviation of Within Row Spacing

Of the eight plots evaluated in 2007, two showed a significant linear fit R value at the 0.01 α level for individual plant spacing (Table 5). These locations were Warren County, KY plot three, where R = 0.40 (n = 117) (Garst 83-97) and Caldwell County, KY plot number one, where R = 0.38 (n=133) (Pioneer 33M54) (α = 0.05). These R values indicated an increase in an individual plants grain yield as the distance between it and its immediate neighbors increased. Using $Y\sigma = Y_0 - c\sigma$, where c is the slope of regression, Y_0 is the yield when σ is less then 5.1 cm, and Y_σ is the grain yield in kg/ha at a given σ only Caldwell County plot number two had a significant linear fit R value (Significant at 0.05, r = -0.82, n = 5),. $Y\sigma = 17{,}036 - 376.54\sigma$ describes the grain yield loss due to uneven spacing for Caldwell County plot number one.

Of the seven plots evaluated in 2008, four plots showed a linear R value of 0.05 significance for individual plant spacing. Using $Y_\sigma = Y_0 - c\sigma$ WKU AREC plot one (Crows 5176S-RR) was significant at the 0.05 level (R = -0.81, n = 5). Here $Y_\sigma = 11{,}748 - 134.12\sigma$ (Y is expressed in kg/ha).

The individual plant grain weights of the spacing plots can be fitted to the Duncan yield curve, relative to their distance to their closest neighbors. This requires the mean distance to the plants' neighbor be converted into a population density. For example a plant 9.5 cm away from each of its closest neighbors on 76.2 cm wide rows has a local relative population of 138,700 plants ha^{-1}. Using this style of transformation the local population of each plant in the spacing study was calculated. The highest R value of these transformations in 2007 occurred at the WKU AREC plot 4 (Garst 83-53). Here the R value was -0.23 (n = 92) and is significant at the 0.05 level of confidence. In 2008 WKU AREC plot three was significant at the 0.01 level with an R value of -0.54 and (n = 146). Medium flex, medium population plot in 2008 was also significant at the 0.01 level with an R value of -0.53 with (n = 114). To be able to fit the individual plant data to an exponential decay curve all zero value (no grain) plants were discarded. Figures 5-15 display the relative population density of plants and the grain yield on Duncan's yield curve, individual plant spacing linear fit, and σ value linear fit for plots which are significant at the 0.05 level. These charts display the great variability when observing individual plants in both the linear increase in space and the Duncan's yield curve applied to individual plants. Table 5 displays the significant R values for individual plant spacing, σ of within row spacing, and Duncan’s yield curve applied to individual plants.

Table 5. R Values of 0.05 significance or greater for individual plant spacing (linear), standard deviation of within row spacing (linear), and relative local plant population density (exponential) effect on grain yield.

Year	Location	Cause	R	n
2007	WKU AREC Plot 3	Plant Spacing	0.40	117
2007	C. County, KY Plot 1	Plant Spacing	0.38	133
2007	C. County, KY Plot 2	σ	-0.82	5
2007	WKU AREC Plot 4	Population Density	-0.23	92
2008	WKU AREC Plot 3	Population Density	-0.54	146
2008	WKU AREC Medium Flex Cultivar	Population Density	-0.53	114
2008	WKU AREC Plot 4	Plant Spacing	0.24	148
2008	WKU AREC Plot 3	Plant Spacing	0.33	143
2008	WKU AREC Plot 2	Plant Spacing	0.44	136
2008	WKU AREC Plot 1	Plant Spacing	0.24	132
2008	WKU AREC Plot 1	σ	-0.81	5

Figure 5. Individual corn plant grain production of cv. Garst 8397 at WKU AREC as influenced by mean distance from neighboring plants in 2007.

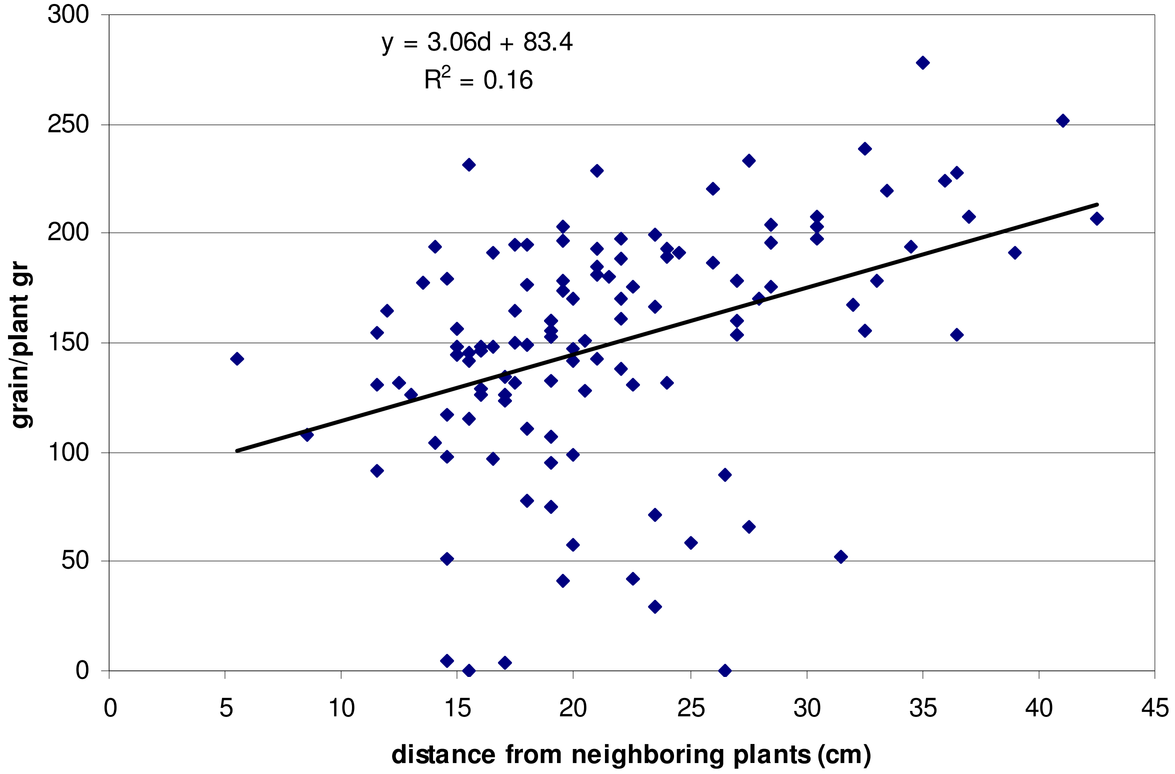


Figure 6. Individual corn plant grain production of cv. Pioneer 33M54 in Caldwell County, KY as influenced by mean distance from neighboring plants in 2007.

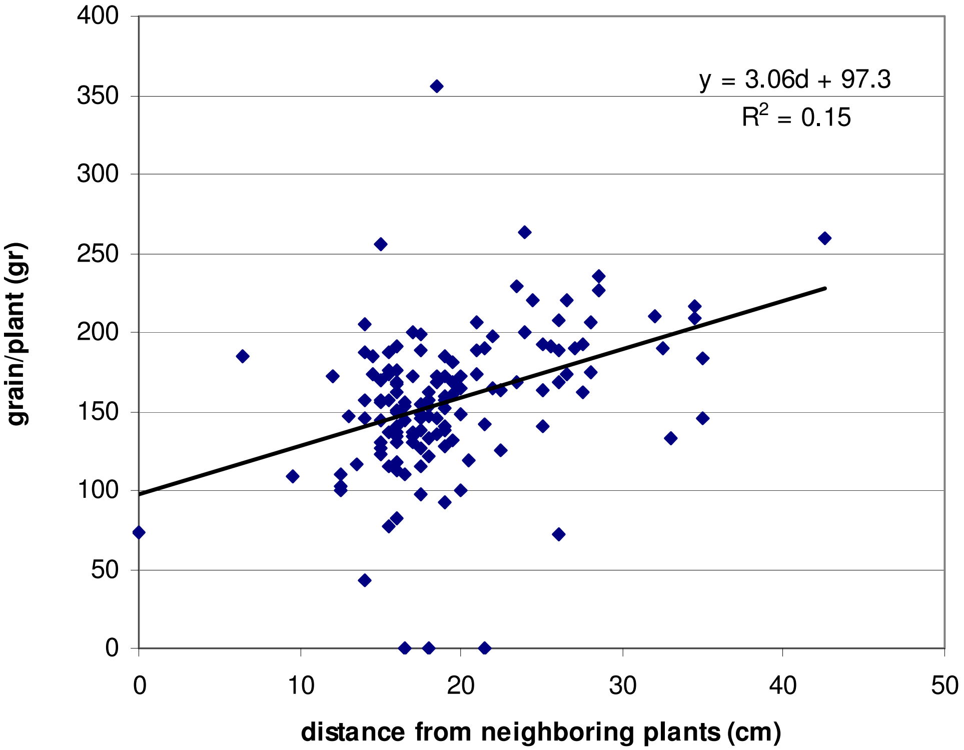


Figure 7. Individual corn plant grain production of cv. Crows 5176S-RR at WKU AREC as influenced by mean distance to neighboring plants (Plot 1) in 2008.

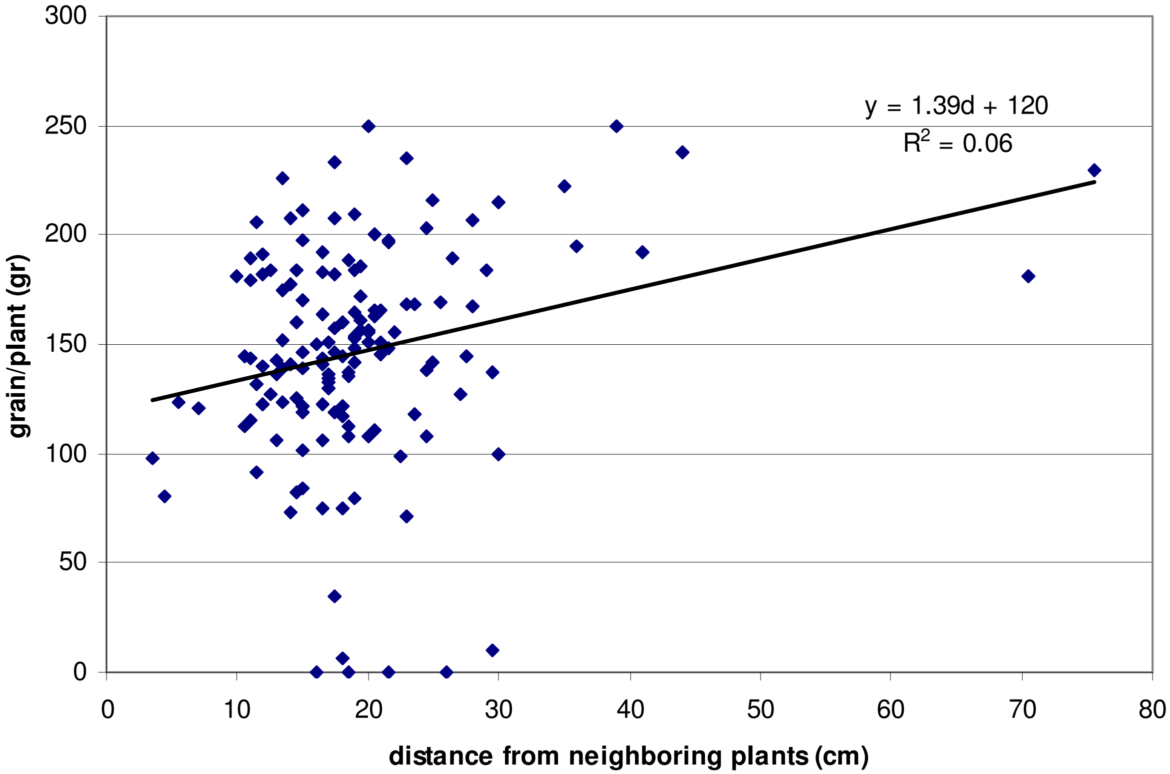


Figure 8. Individual corn plant grain production of cv. Crows 5176S-RR at WKU AREC as influenced by mean distance to neighboring plants in 2008 (Plot 2).

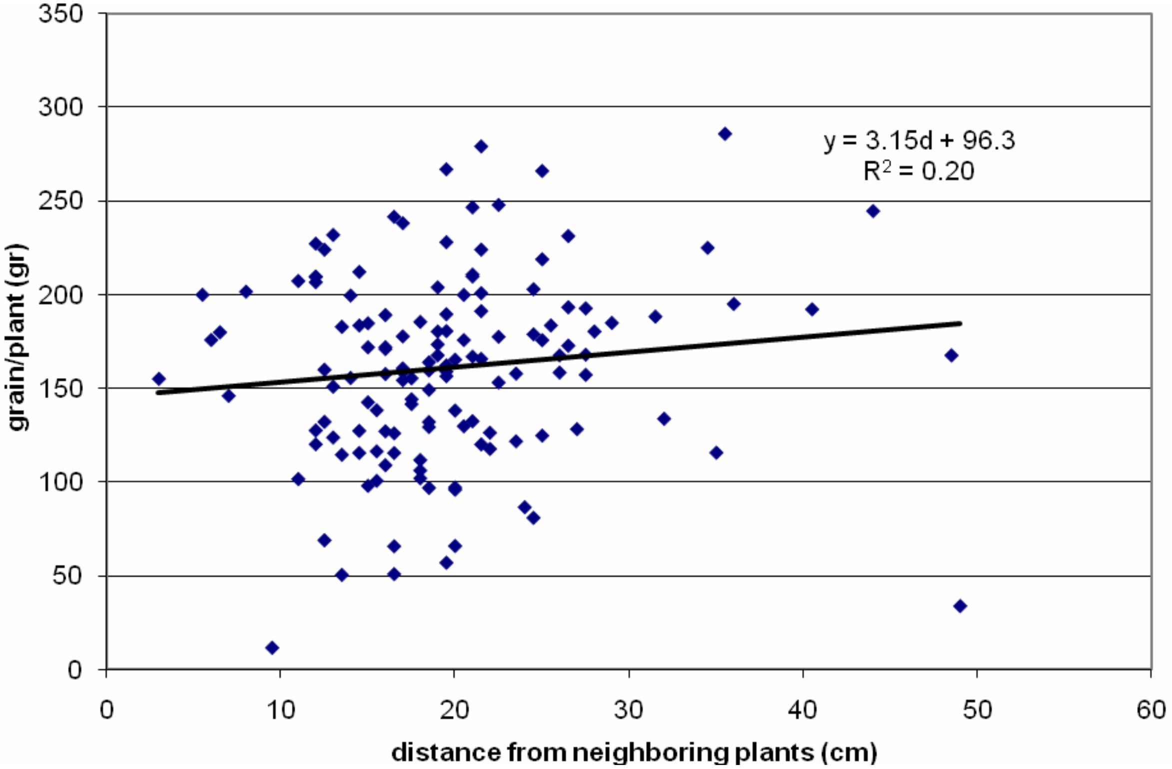


Figure 9. Individual corn plant grain production of cv. NK N72-Q6 at WKU AREC as influenced by mean distance to neighboring plants in 2008 (Plot 3).

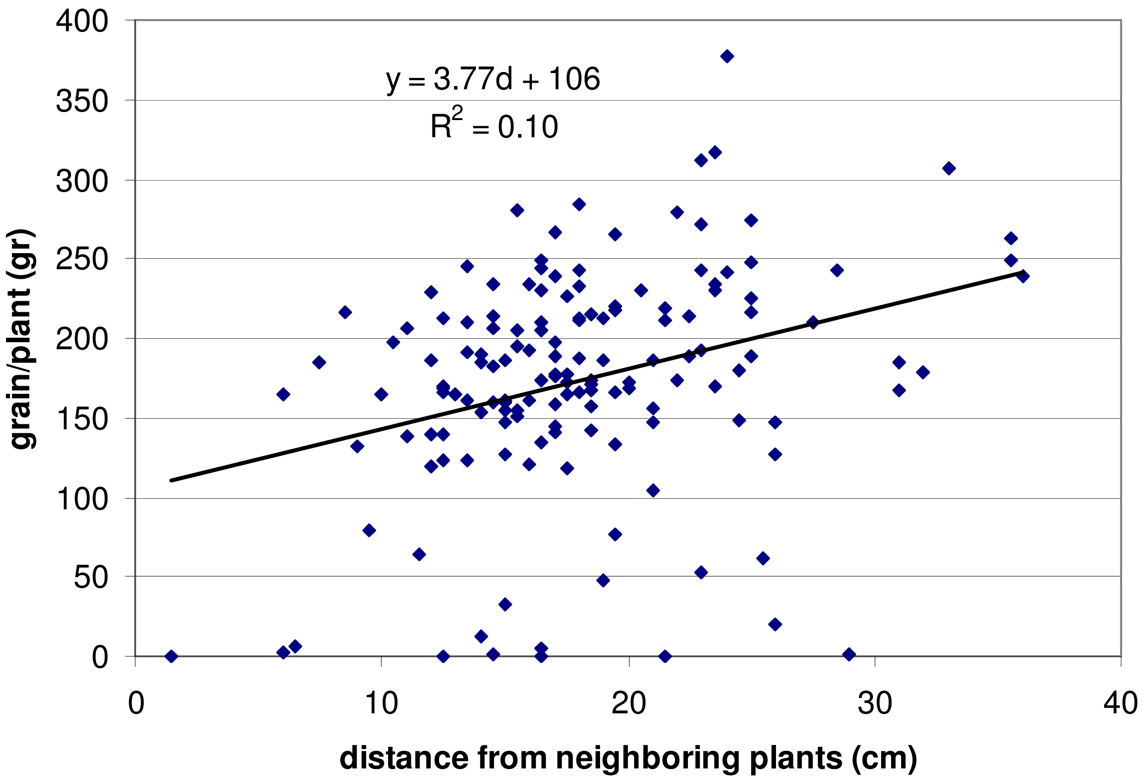


Figure 10. Individual corn plant grain production of cv. Syngenta NK N72-Q6 at WKU AREC as influenced by mean distance to neighboring plants in 2008 (Plot 4).

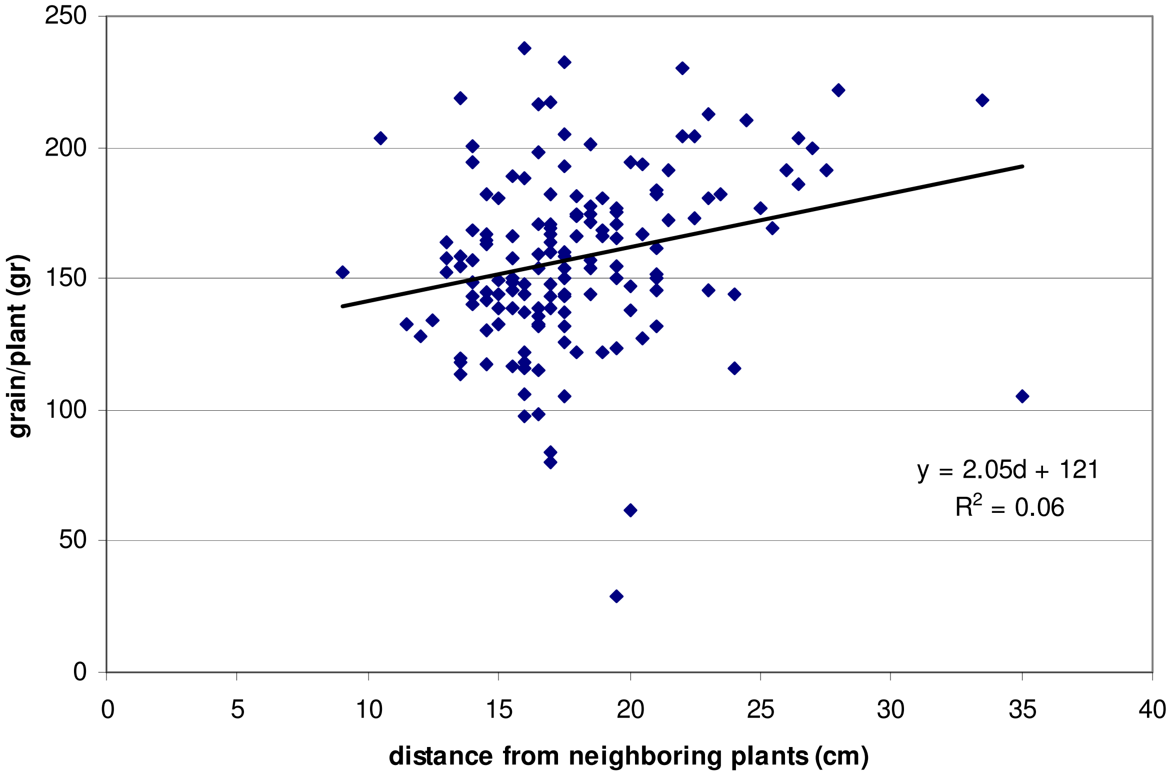


Figure 11. Standard deviation of within row spacing effects on corn grain yield in Caldwell County, KY (Plot 2) in 2007.

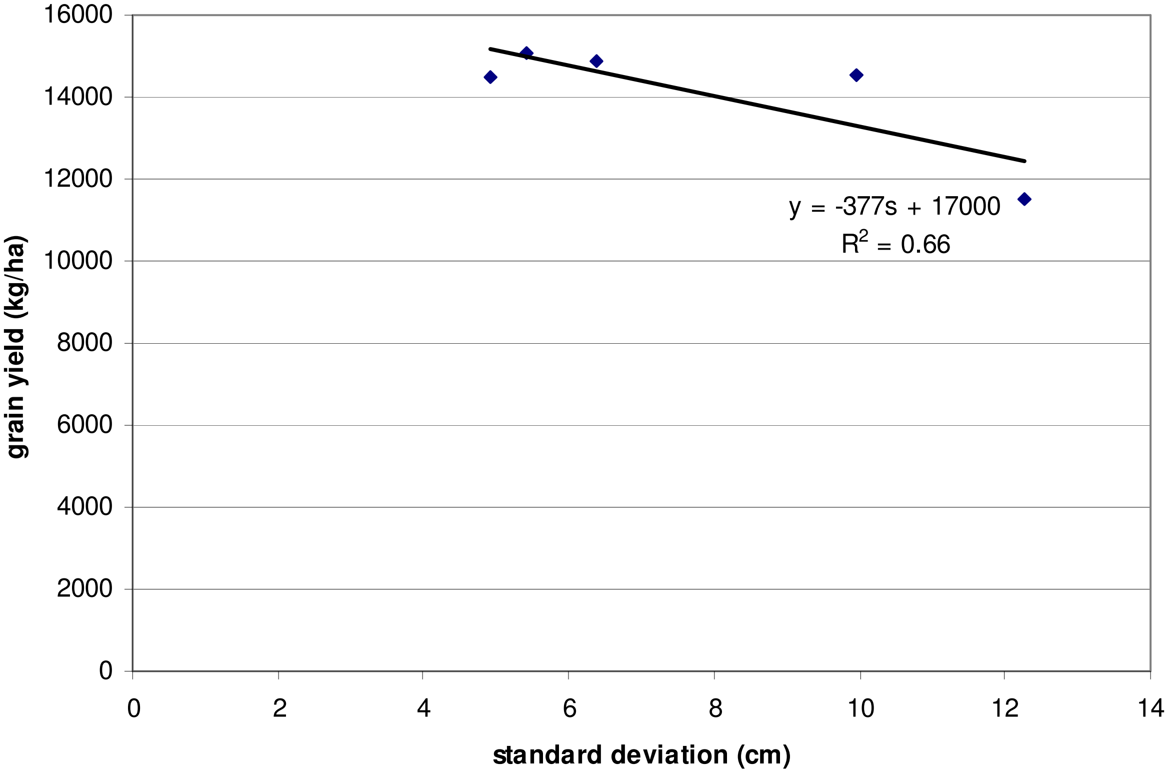


Figure 12. Standard deviation of within row spacing effect on corn grain yield at WKU AREC in 2008 (Plot 1).

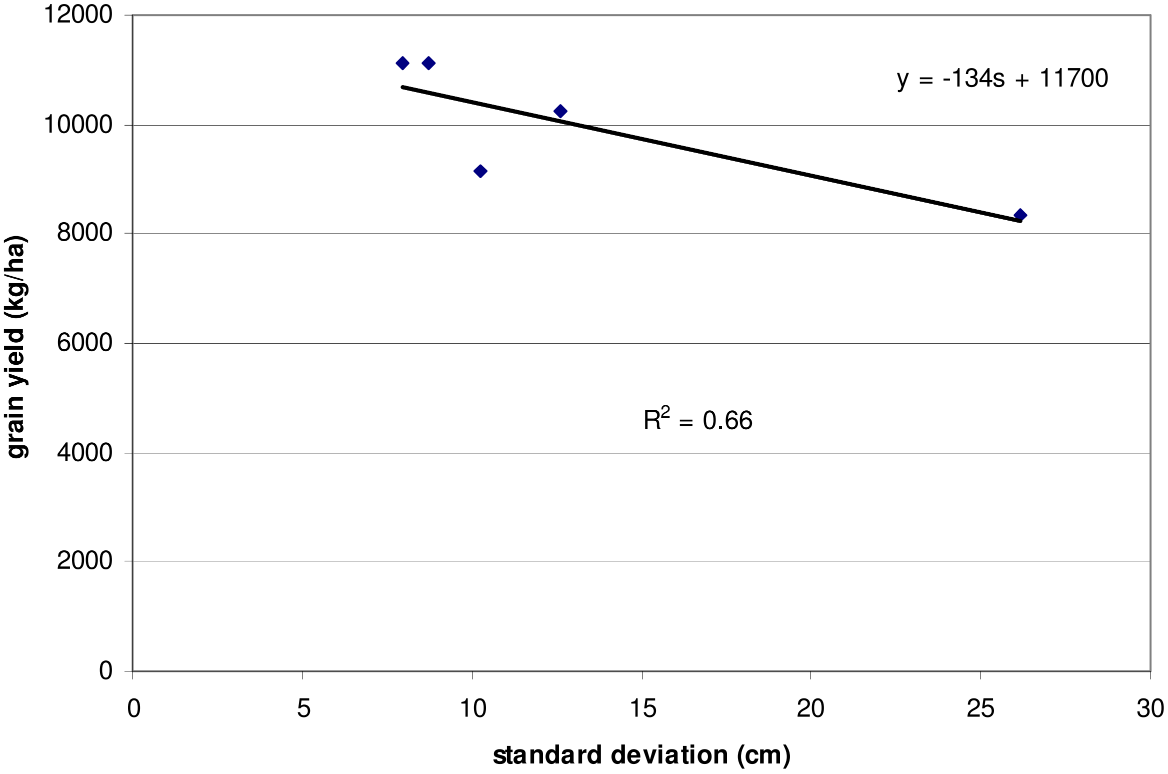


Figure 13. WKU AREC plot 4 individual corn plant grain production fit to Duncan yield curve in 2007.

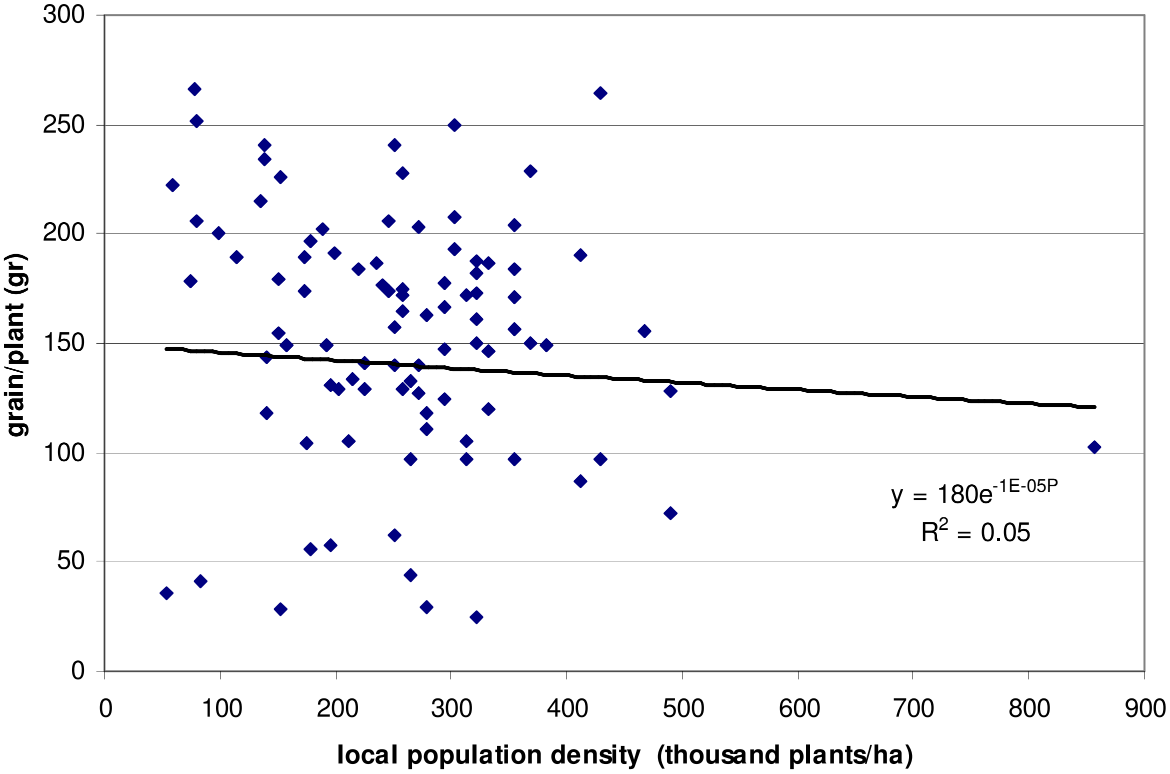


Figure 14. WKU AREC plot 3 individual corn plant grain production fit to Duncan yield curve in 2008

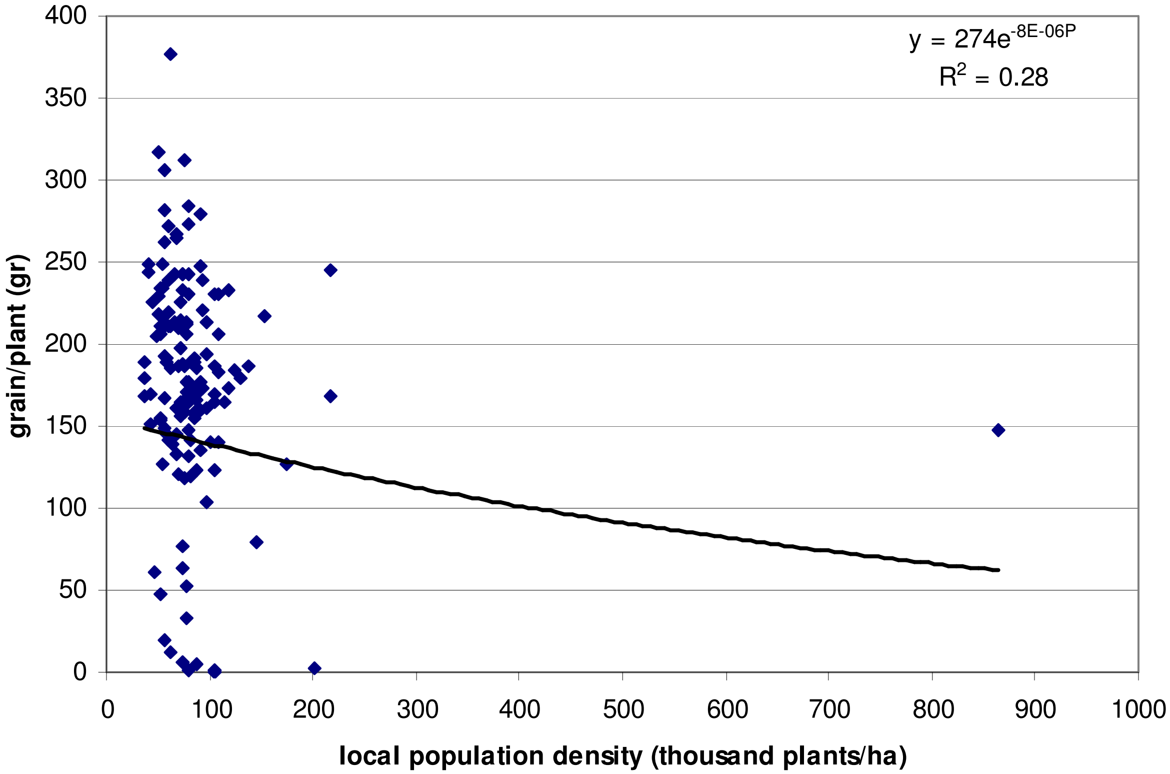
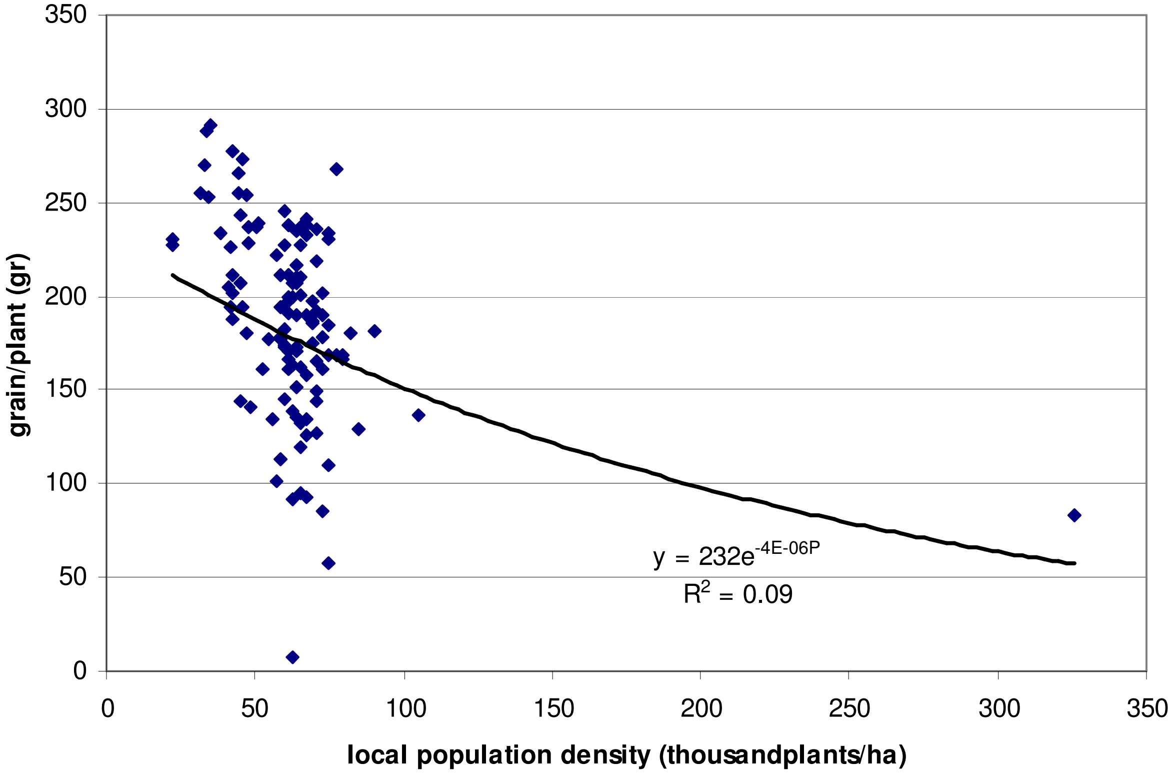


Figure 15. Medium Flex cultivar seeded at 61,000 seeds ha^{-1} with individual corn plants fit to Duncan yield curve in 2008.



Chapter 5

SUMMARY

The Duncan yield curve and the Russell aboveground biomass model fit all 6 genotype by environment interactions for 2007 and 2008 to the $\alpha = 0.05$ level of confidence when evaluated over a 5.3 meter length on 76.2 cm wide rows. Individual plants fit linearly at $\alpha = 0.05$ in 9 out of 15 plots. Individual plants fit the Duncan yield curve at $\alpha = 0.05$ in 3 out of 15 plots.

The linear r value for standard deviation of within row spacing's effect on grain yield was negative for 14 out of 15 spacing plots, however, R was significant at $\alpha = 0.05$ in only 2 of those 15 plots. Some of the 13 plots which did not fit significantly had little variation in σ values between rows. To further evaluate the effect of uneven spacing forced unequal spacing is needed.

Both the Duncan and Russell models fit during the conditions of 2007 and 2008 at WKU AREC. These years had less than average rainfall for Bowling Green, KY (Appendix 4). Further study is needed to determine the optimum planting density for grain yield during years with higher rainfall. Applying the Duncan grain yield model with a N fertility study could also reveal N effect on the optimum planting density for a cultivar. The influence N levels have on the optimum planting density of a cultivar would demonstrate the N efficiency of the tested cultivar. Further study is also need to explain the variation in individual plant grain production.

Appendix 1

Mathematical models for maize

I. $y = ae^{-bP}$	**I. $Y = aPe^{-bP}$**
Grain yield per plant at plant density P	**Grain yield per unit of area**
I_a. $\ln(y) = -bP + \ln(a)$	Ia. $\ln(Y) = -bP + \ln(aP)$
II. $\mathbf{m} = \frac{1}{BP + A}$	**II.** $\mathbf{M} = \frac{P}{BP + A}$
Total aboveground biomass per plant at plant density P	**Total biomass per unit of area**
II_a. $\eta = \frac{1}{m} = BP + A$	II_a. $N = \frac{1}{M} = \frac{A}{P} + B$
η is the number of plants necessary to produce a unit of aboveground biomass.	N is the area necessary to produce a unit of biomass.
III. $s = m - y$	III. $S = M - Y$
Stover yield per plant at plant density P	Stover yield per unit of area
IV. $H = y/m$	IV. H is the same.
Harvest index at population P	
V. $f = s - y*r$	V. $F = S - Y*r$
Fodder yield per plant	Fodder yield per unit of area

Multiplying any left side equation by P gives the mass per unit area. The result is denoted as a capital letter.

VI. $E = Y*V - \mu S_P - {'\Omega}$:
Economic gain (net profit)

1. P is the area plant density.
2. a and b are the parameters of the grain yield equation.
3. A and B are the parameters of the above ground biomass equation.
4. r is the ratio of cob mass to grain mass.
5. η and N are the biomass plant density quantities (new terms in agronomy recommended by M. W. Russell).
6. $'\Omega$ are the fixed costs
7. μ is the cost of seed
8. S_P is the number of seeds needed to reach a germinated population of P
9. y_m is the grain yield per plant at the plant density that produces the maximum yield per unit of area.
10. Y_m is the grain yield per unit area at the plant density that produces the maximum yield.
11. m_m is the aboveground biomass per plant at P_{max}.
12. M_m is the aboveground biomass per unit area at P_{max}.
13. Any subscript n denotes a maximum aboveground harvest index.

Appendix 2

2007 Soil Fertility Recommendations for 11000 kg/Ha Yield Goal.*

Soil pH	6.8	Cation Exchange Capacity	Recommendations
		$\frac{9.0meq}{100gr}$	
Phosphorus	94 kg ha^{-1}	Cation Saturation	74 kg $ha^{-1}P_2O_5$
Potassium	448 kg ha^{-1}	%K 5.4	45 kg $ha^{-1}K_2O$
Calcium	3656 kg ha^{-1}	%Ca 71.6	
Magnesium	506 kg ha^{-1}	%Mg 19.3	
Sulfur	25 kg ha^{-1}	%H 3.0	11 kg $ha^{-1}S$
Boron	.9 kg ha^{-1}	%Na 1.0	1.3 kg $ha^{-1}B$
Copper	8.5 kg ha^{-1}	K:Mg ratio .27	
Iron	215 kg ha^{-1}		
Manganese	715 kg ha^{-1}		
Zinc	7.6 kg ha^{-1}		
Sodium	45 kg ha^{-1}		
Organic Matter	1.5%		

*N was applied at 224 kg ha^{-1} in 2007 and the P was also lowered to 62 kg ha^{-1}.

B was not applied.

Appendix 3

2008 Soil Fertility Recommendations for 10,700 kg/Ha Yield Goal*.

Soil pH	6.5	Cation Exchange Capacity $\frac{7.3meq}{100gr}$	Recommendations
Phosphorus	202 kg ha^{-1}	Cation Saturation	34 kg $ha^{-1}P_2O_5$
Potassium	401 kg ha^{-1}	%K 5.9	45 kg $ha^{-1}K_2O$
Calcium	2934 kg ha^{-1}	%Ca 70.9	
Magnesium	302 kg ha^{-1}	%Mg 14.2	
Sulfur	27 kg ha^{-1}	%H 7.5	11 kg $ha^{-1}S$
Boron	1.8 kg ha^{-1}	%Na 1.4	.7 kg $ha^{-1}B$
Copper	6.7 kg ha^{-1}		
Iron	242 kg ha^{-1}		
Manganese	627 kg ha^{-1}		
Zinc	15.9 kg ha^{-1}		
Sodium	54 kg ha^{-1}		
Organic Matter	1.4%		

* N was applied at 224 kg ha^{-1} in 2007 and the P was also lowered to 62 kg ha^{-1}
B was not applied

Appendix 4

Rainfall monthly totals at WKU AREC (Mesonet)

Month	Precipitation (cm)		
	2007	2008	30 year average*
May	2.84	13.6	10.67
June	15.9	4.39	10.16
July	3.89	14.6	10.67
August	2.87	1.63	9.14
September	2.97	5.00	7.87

*city data (2006)

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