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# Zeno of Elea: Where Space, Time, Physics, and Philosophy Converge *An Everyman's Introduction to an Unsung Hero of Philosophy*

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# Zeno of Elea: Where Space, Time, Physics, and Philosophy Converge

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## Abstract

*Zeno of Elea, despite being among the most important of the Pre-Socratic philosophers, is frequently overlooked by philosophers and scientists alike in modern times. Zeno of Elea's arguments on have not only been an impetus for the most important scientific and mathematical theories in human history, his arguments still serve as a basis for modern problems and theoretical speculations. This is a study of his arguments on motion, the purpose they have served in the history of science, and modern applications of Zeno of Elea's arguments on motion.*

Keywords: *Achilles, Arrow, Dichotomy, Eleatic, motion, paradox, Parmenides, pre-Socratics, Stadium, Zeno of Elea*

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## Foreword

When approaching a paper regarding Zeno's paradox in such an encompassing manner, an explanation of the motives of this work, as well as some preliminary comments related to the understanding of this work as a fluid whole, are in order. There are several themes which I wish to convey here, and I am concerned that without the proper guidance these themes may not properly manifest themselves on account of the eclectic nature of the paper.

The first question one will ask after reading this paper is most likely "Do you seek to disprove the existence of continuous motion?" To make such an assumption upon digestion of the paper is one most merited. The answer, however, is frankly no.
Most of us have faith that we live in a universe in which the force through which things progress and change occurs is continuous. The word I wish to stress here is *faith*, which leads to the first of my themes in this paper. In a society where the basis for truth becomes more reliant upon the evidence and the "thumbs up," so to speak, from the natural sciences – currently biology, geology, and physics in particular – and less on faith – religious, optimistic, or otherwise – I seek to provide evidence that even an idea such as continuous motion that is part of the very foundation of what man believes to exist, that even this is not solidly proven and is but a theory based upon our own perceptions, and

thus it is subject to the scrutiny to which any other theory is susceptible. If through faith one accepts the perception of continuous motion at face value, it follows that a sullied notion such as faith may be given new relevance and importance in a world which often seeks so diligently to discard it, even when science does offer evidence to the contrary. My reasoning for the necessity of a call to faith as the basis of continuous motion may be understood in my section on Psychology.

Indeed, man now has the luxury of science. Though science has accomplished great things and allowed for the progression of society and technology – most likely more so than any other contrivance of the human mind – the problem which faces science is the fundamental flaw of the scientific method: for two reasons it is ultimately self-defeating. The nature of the scientific method, and indeed of any formal, empirical inquiry, is one that will never allow a complete account of the universe. As technology and knowledge of the universe become more acute, man discovers that there is even more which he does not know, or that he has not discovered, than he had sometimes ever imagined previously. This point is best emphasized in Nietzsche's *The Birth of Tragedy*:

> *But science, spurred by its powerful illusion, speeds irresistibly toward its limits where its optimism, concealed in the essence of logic, suffers shipwreck. For the periphery of the circle of science has an infinite number of points; and while there is no telling how this circle could ever be surveyed completely, noble and gifted men nevertheless reach, e'er half their time and inevitably, such boundary points on the periphery from which one gazes into what defies illumination.* (p.97-98)

We furthermore find that science itself admits in a preliminary disclaimer that it may never reach beyond the bounds of material existence; notions such as faith and the soul are fundamentally transcendent of science's inadequate clutches. Another abstract construct which science may not lawfully ponder, though it is often never considered, is the infinite. All scientific investigation must begin with empirical evidence – from some experience of the senses. Because one cannot experience infinity, it is thus an abstract construct completely outside the bounds of the material universe or of the natural sciences that serve as a methodology for its discovery and understanding. So then one must accept that anything which involves faith, a soul, or the infinite – namely God, religion, and faith – is intrinsically something with which science must refuse to deal.

One may ask "but doesn't the science of mathematics deal quite commonly with the infinite, with its various formulas and theorems?" Indeed mathematics does commonly dabble in infinity; however, we must be careful as to what we call a science. Mathematics is scientific in the sense that its theorems are hypothesized, tested, and established. However, mathematics is fundamentally different from any of the physical sciences with which it is associated. Mathematics is quite unlike biology, chemistry, physics, and the like; rather than serving as a study of the physical world, as these are, mathematics is a formalization of principles of logic. These principles of logic, unlike elements of the physical world, are not subject to variation or change. As many have heard asserted, two and two have always been equal to four, and will continue to be forever. Mathematics, rather than being a study of physical reality, is the tool that represents logical reality, which we use to study reality's physical manifestations. This view of mathematics borrows heavily from Plato's view of mathematics in that it takes an ascendancy over the world of "seeming" and is rather a part of the eternal, intelligible world. Though I am skeptical of the Platonic tradition, I am nonetheless confident that this is an accurate view of mathematics, and being so it is necessary to make this distinction for the reader.

Should science seek to prove or disprove such things on its own grounds, an informal fallacy occurs – the fallacy of ignorance. A fallacy of ignorance occurs when one concludes that an argument is false because it cannot be proven, or that it is true because it cannot be disproved: it takes only elementary deductions then to understand why science is in error any and every time it seeks either to prove or disprove the existence of a creator, a personal god, etc. which by science's very nature it can never prove or disprove. Nonetheless, many armchair scientists with little understanding of science's objections and limits press on in error in attempts to disprove these supernatural hypotheses for the paradigms of existence in the name of scientific atheism – an atheism that through science alone cannot have any logical foundation.

In the spirit of such argumentation, another purpose of this work subsequently becomes imminent. Not only in the present times, but through all time have people been subject to the competing views of theologies, philosophies, and innumerable mundane arguments. Though some of these arguments may only be reduced to aesthetic ideals or matters of opinion, an equally abundant number of such debates may have clearly defined winners, should one appeal to reasonable and logical faculties that are intrinsic within all people. For example, though one may never be able to satisfactorily end the debate between science and religion for one side or the other, as the ultimate questions involve compelling yet aesthetic requirements of faith, one can decide which arguments for or against are reasonable and logical, and likewise those which are irrational contrivances appealing to the perpetuation of either side's ambitions and ideals rather than appealing to reason or solid evidence.

This work is a micro-study of a series of arguments. A set of arguments, reasoned judgments of whether these arguments are sound or unsound, and the explanations behind these judgments are all explored. I hope that analysis of these arguments may to some degree strengthen the logical and reasoning faculties of the reader, that the reader may translate such strength into his or her own life when faced with more practical arguments in the "common-sense" world. No person is inhibited by keener powers of reason.

These are times when the very things which man has believed to be true and existent are to be challenged. In light of the intrinsic uncertainty[1] and relativity which the currently accepted scientific and philosophical theories provide, nothing stands firmly held sacred by the minds of those who seek an integrated theory of our universe. Though I have explicitly expressed that I hold faith in the existence of continuous motion, at least the common colloquial sense in which we experience it, it is quite possible that the arguments and evidence provided here may change the reader's perspective on the nature of the universe. Indeed, such a goal is not primary, but nonetheless I may only hope that I challenge the reader to at least reconsider what he or she believes to be real in the general sense, and then after weighing such evidence, reason whether or not such considerations are in fact provable or are but mere reliance.

With the study of argumentation, and the controversies of quantum physics, faith,

[1] The uncertainty to which I refer is in the general sense, that, especially under present conditions, scientists are unsure as to the true nature of matter and energy; because of such uncertainty, many opposing theories have spawned, each about as possible as the other. This uncertainty, however, is not to be confused with the acceptance of the Heisenberg Uncertainty Principle. The ramifications of such a mistake are discussed in the heading *The Philosophy of Weird Science*.

and other strange bedfellows aside, one final purpose becomes manifest. No definitely-comprehensive account has been afforded to Zeno since Aristotle's first refutations in the Fourth Century B.C. Yet Zeno has been offered at least a passing fancy in, if not a nod of admiration for, his paradoxes and/or Eleatic philosophy by almost every Western philosopher of notoriety since Plato. Kant, for example, in his logic textbook asserts that Zeno "distinguished himself as a man of great intellect and acumen and as a subtle dialectician (p.32)." As far as Eleatic philosophy is concerned, not only did it have great effect upon the entire Platonic tradition of Western thought (including Augustine, Descartes, and a host of others – all of which are discussed in a later section), it also served as an inspiration to Leucippus (Russell, 64) and Democritus, the co-fathers of atomic theory. Indeed this very theory was preliminary to the scientific discovery of the atom and the atomic theories following from it, which in no small part launched the phenomenon of intellectual discovery and rigor that has guided science for centuries. We also find that one of Socrates' (through Plato) most significant contributions to philosophy was his method of dialectic, which Bertrand Russell cites as first being implemented by Zeno in his arguments (*The History of Western Philosophy*, p. 92). So here we find Zeno not only as a thinker esteemed by the overwhelming preponderance of the great Western philosophers, but even as the "man behind the curtain" of some of the foundational discoveries and theories which have guided Western academia into the present. To understand and argue the thought of such a man is a task most honorable and propitious in an age dominated by edicts of science: one which is becoming increasingly acquiescent to radical theories and hypotheses.

## Introduction

Man has always placed divisions between his categories of learning. Presently, human knowledge exists on something that may be compared to a continuum, yet with a strict division between the poles of this continuum – a divide that is based upon Aristotelian principles. On one side of this continuum are what Aristotle called in his *Posterior Analytics* "scientific knowledge" (Gr. *epistêmê*) and "art" or craft (Gr. *technê*). Scientific knowledge is gained by coming to understand nature by human experience, experience that is resultant of impressions from the environment being accumulated in memory. The formal study of this empirical wisdom is the foundation of natural science. Art is the application of the knowledge of a trade or science to some practical skill (which may be any skill: pottery, medicine, law, mechanics, etc.). It is the spirit of scientific knowledge and craft that is salient among modern science and formal education in general. Most academic subjects are extensions of *epistêmê* and *technê*: history, economics, psychology, and the entire host of academia. Champion and central among these sciences, the science which seeks most primarily to understand the nature and relationship of the matter and energy that compose the sensible universe, is physics. On the other side of this continuum exists that which lies in the hypothetical, the discussion of what is basic to all the experiences of humanity – what Aristotle defined as understanding of the first principles (Gr. *nous*). This dichotomy of academic subjects into the empirical and the speculative, the practical and the theoretical, the utilitarian and the aesthetic has been seen for millennia, being found in the most ancient of Western philosophical texts. Sadly, most of academia has succumbed to the notion that the highest goal for human knowledge is to find causation and effect, to understand what causes bring about what effects and why, without an integrated understanding of knowledge writ large, or of human nature. Perhaps the last intellectual garrison for *nous*, academic philosophy, presses on diligently in a hope to avert the firestorm of superficial knowledge from squelching the importance of *nous* from higher education entirely.

Scholars who adhere to one of these two methodologies of knowledge tend to not get along with one another intellectually, and those who study solely in one extreme or the other tend to exhibit a great deal of intellectual bias. Indeed, there are those who promote the idea that understanding underlying principles is a heady subject for aimless philosophers, those whose heads scarcely come down from the clouds to contend with what matters, that is, practical application; likewise there are those who find members of the previous group lacking vision, feeling that they are shallow and without concern for the ultimate knowledge of fundamental principles that may allow the human race insight into the great questions of nature and of existence itself. Such a contention surely sounds exaggerated and unrealistically haughty. One may be compelled to ask oneself "is such a seemingly "tribal" type of conflict honestly occurring in universities and academic circles?" The reality, as this author has experienced as a student, is that sadly it does.

However, one finds that occasionally the lines between these two extremes can blur. Current theories in physics, such as relativity and quantum mechanics, are many times no different from philosophy in either appearance or substance. Throughout the history of thought there have there been those who walk upon these blurry lines, simultaneously delving into both the physical and philosophical modes of knowledge. The most brilliant of those integrators have contributed significantly to both sides of the chasm. One of those who walked this precarious path was Zeno of Elea, a Greek philosopher who lived some two and a half millennia ago. This lone genius devised a set of arguments, of which four will be examined. These four brilliant arguments, well over 2,000 years old and devised before the innovation of any modern conceptions of mathematics or physics, have continued to provoke questioning even when restated using modern physical descriptions of space and time. This study is comprised of several sections, which involve not only the presentation of Zeno's paradox and its scientific and logical solutions but also the various implications across multiple disciplines to which his paradox is applicable. Claims supported by evidence are given to argue against some of the most fundamental preconceptions of human existence, but with a forewarning not to necessarily accept everything presented as truth. This is a short expedition through logic and science, and an adventure into some of the very pillars of human belief and existence.

## History

In order for one to understand that which spawned Zeno's conception of his paradoxes, it would be beneficial to understand the history of the school of thought in which he was educated and the philosophy of his teacher and mentor. If one is able to understand the intellectual climate during this time in Greek history, one may have a clearer view of the ingredients which were placed into Zeno's training and the product Zeno produced when given these ingredients.

The line of thought, which eventually led to the Eleatics and Zeno's arguments, began circa 570 BC at the dawn of Greek philosophy. Among these "fathers" who were laying the groundwork for Western philosophy through their contrived cosmologies and theologies was Xenophanes of Colophon. Xenophanes, who was exiled and thus forced to become a wandering nomad, is widely considered by Greek scholars to be the most eccentric and unconventional of the early philosophical poets[2]. Concurrently, scholars are also in accord that Xenophanes formed the most scientifically inaccurate early view of the cosmos. Even ancient scholars were highly critical of him; Heraclitus described Xenophanes as "one of those whom learning had not taught sense" (Boardman, Griffin, & Murray, 1998, p. 133). What quaint propositions could merit such derision from both ancient and modern critics? Xenophanes believed that the earth extended to infinity in all

[2] It is difficult to ascertain that the early pre-Socratics were "philosophers" in the sense that the thinkers following Socrates were. Rather, these men were poets with ideas that inspired the beginnings of academic philosophy, and are thusly given merit as the first philosophers.

directions, that the sky was infinitely high, and that there were infinitely numerous suns so that a different one was seen in the sky each day: indeed quite dissonant with the modern scientific consensus.

What is to be noted here, however, was Xenophanes' theology, which did attract the attention of other ancient thinkers. Xenophanes is accredited with having been the first of the philosophers to make an attempt to devise a methodical account of the nature of God (Lesher, 2002). Xenophanes is also one of the earliest Greek proponents of monotheism. The god of Xenophanes was universal and singular, "formless." Not only did it have no definite shape or form, it remained eternally static yet effortlessly moved the universe by virtue of its immense intellect. (Boardman, Griffin, & Murray, 1998)

It may also be noted that Xenophanes could very well be credited as one of the earliest contributors to the philosophy of mind, in modern terms "psychology." In his poetic discourse on the nature of God, he points out that various tribes worship gods who look like them, and goes so far as to say, "if cows and horses had hands they would no doubt depict their gods as cows and horses" (Boardman, Griffin, & Murray, 1998, p, 132). Prior to Xenophanes, no one voiced the notion that man arranges his environment – whether consisting of gods or otherwise – to best suit his personal/cultural interest. This statement is one still viable in theological arguments about the existence and nature of God.

Despite Xenophanes of Colophon's nomadic milieu, he eventually settled in Elea, where a school based upon his philosophy was founded. The founder and greatest philosopher of this school was Parmenides, who was born around 570 B.C. and was active as a philosopher by 539 B.C. (Boardman, Griffin, & Murray, 1998). Parmenides is considered among the most important of the pre-Socratic philosophers, namely for his contributions to logic and scientific theories. His philosophy provided the framework for Plato's philosophy, particularly the idea of an unchanging reality transcendent of the world of seeming. The great difference between the two, however, is that Plato did not reject the notion of plurality as Parmenides did. Nonetheless, Plato refers to Parmenides as a personal "father" (241d) in his dialogue *Sophist*. It is clear then to see that one need not look with much diligence to discover that Parmenides may be called the grandfather of Western Philosophy: for from Parmenides came Plato (and likely Socrates), who continued the tradition of the philosophy of being, which states that there are static, underlying principles (or for Parmenides an underlying thing, since he does not believe in plurality) that transcend the illusionary world of seeming and change. From Plato came Augustine, from Augustine came Descartes, and in light of such a lineage both the Catholic religion and rationalistic philosophy owe a great debt to this easily forgotten pioneer.

Parmenides not only developed the school's doctrine of unchanging oneness formally, but he is also accredited as being among the first philosophers to use deductive logic in argumentation. Parmenides' doctrine of a static world behind the material one was evolved by Plato into what became the cornerstone of his philosophy – and the first of the two philosophies (along with Aristotle) that have since dominated Western thought – the world of the forms. Furthermore, he continued with Xenophanes' other significant contribution to the evolution of Western thought: that one could not begin to explain or understand the nature of that which is within the bounds of sensible human experience – only matters of the intelligible may be comprehended. In the realm of Greek philosophy, no one even dared challenge the tenets of this argument before Aristotle; clearly, Aristotle and those of his tradition have to this day been unable to satisfactorily answer this argument for all. Because of this Xenophanes and Parmenides are considered to be the earliest important contributors to the evolution of the scientific method. The empiricist philosophy, concurrent with the progression of modern science, owes quite a debt to them as well in two ways. Firstly, the early development of this method was of the tradition of Francis Bacon, who undoubtedly had it in mind when he pioneered the scientific method. Secondly, it is the Eleatic distrust of the senses that has led to the tradition opposing Empiricists, Rationalism. As Empiricists emerged as a reaction to the Rationalist tradition, it is from this distrust of the senses the Empiricists have counter arguments to refute while providing evidence for their position. One of Parmenides' greatest students, our beloved Zeno, clearly had a rather well-constructed platform upon which to construct his arguments.

Parmenides' severe rejection of any competing philosophy of the time subjected him to a great deal of scrutiny from the rest of the philosophical world. These champions of Greek convention used rigorous deductive logic, Parmenides' own creature, as a weapon by which to destroy each tenet of Parmenides' philosophy. Zeno, a loyal and zealous student, desired to show the Hellenic world just how absurd taking logic to such extremes was by using that very tool to prove that the Eleatic philosophy was indeed true, and that through logic one might seek to either prove or disprove anything to the point of absurdity. The method of argumentation which Zeno utilized may be found in the section entitled "Zeno's Logic."

## Paradox

Over the course of Zeno's life, he constructed forty arguments. Of those forty, only eight have survived through the centuries, escaping destruction by conquest and the purging written material in the name of vanquishing heresy. His four on motion are by far the most famous and mulled-over. Zeno made these four arguments to construct a paradox (though this is not the only paradox Zeno constructed, it is synonymous with what is commonly referred to as "Zeno's paradox" in colloquial literature) which would logically prove that motion in the universe, in the way the universe was viewed not only by the common man but by physicists until around the twentieth century, was logically incoherent. These arguments may be divided into two categories: the first two deal with proving that the universe (consisting of space and time) may not be logically infinitely divisible, the second two arguing that a finitely divisible universe is also logically impossible. Should one read Aristotle's *Physics* or many of the works which have since tried to explain Zeno's arguments, he would discover that not only are they difficult at times to follow but sometimes even disagree on exactly what the paradoxes are saying. In spite of this lack of a consensus, this work will try to provide a concise yet easy-to-follow survey of the paradoxes while maintaining strict adherence to the content in their earliest preserved appearance in Aristotle's writing.

### ARGUMENTS AGAINST A CONTINUOUS UNIVERSE

*The Dichotomy*

Zeno's first argument is known as the dichotomy, which will be explained by way of an analogy involving archery. Subject A shoots an arrow at a target. First, it is established that it takes some amount of time, which will be defined as T1, to reach halfway to the target. From between half the time to T (T1) and three quarters of the way to T it takes an amount of time also, which will be called T2. Thus far the time traveled toward T has been T1+T2, and the arrow is three-quarters of the way to the target. One must also understand that T2 is one half of T1. If the process of cutting each "T" in half and adding that total to the original total is continued, assuming the universe is continuous, this may be done ad infinitum. Zeno argues that taking an infinite number of finite amounts of time equals infinity, and though the subject does get closer and closer to the target, the subject still never reaches it. On an even smaller scale, if one wishes to travel even the shortest distance possible, this argument still applies the same: one never travels even the shortest distance possible, just as the arrow never reaches the target. Zeno argues from this that motion can never take place, because one can never actually travel any distance. (Huggett, 2004, Mathpages, 2006, McCartney, 2000)

*The Achilles*

Zeno's second argument, the Achilles, works very similarly to the Dichotomy. It is described with the illustration of a race between the Greek hero Achilles and a tortoise. First, establish that Achilles and the tortoise are moving at different speeds: Achilles at one meter per second and the tortoise at 0.1 meters per second. The tortoise, however, has been given a 0.9 meter head start. Practically one would say that in one second Achilles catches up to the tortoise. However, if one begins to half the distance Achilles must travel in a fashion that is similar to the first argument, one finds that Achilles never catches up to the tortoise. Each time one takes half the distance that Achilles has traveled, one must also take half the distance the tortoise has traveled and add to it the distance between them; the tortoise has traveled a little farther that Achilles must travel in order to reach it. If one continues this ad infinitum, one finds that Achilles must make an infinite number of "catch-ups" to the tortoise, because the tortoise has always traveled a little farther each time Achilles makes his catch-up. Achilles never actually reaches the tortoise. (Huggett, 2004, Rosenstein, 2004)

From these two arguments, Zeno thus concludes that the universe may not be continuous but rather is necessarily constructed of finite particles, because infinitely divisible motion leads to problems such as these. This is important to note, as the entire concept of Zeno's paradoxes lies in systematically proving the logical impossibility of continuous motion in his first two arguments, then proving the logical impossibility of discontinuous motion in his second two arguments.

## ARGUMENTS AGAINST A DISCRETE UNIVERSE

### *The Arrow*

Zeno's third argument is the Arrow. If one assumes that time exists in a series of finitely divisible moments, one may never find a difference between moving and non-moving objects. It is established that at no instant is an arrow in motion, because at any instant the arrow is occupying an amount of space equal to its size – thus the arrow travels no distance. If time, as it was classically considered, is composed of nothing but a series of instants that are of some discrete length, and during none of these moments can one find the presence of motion, then the arrow can never move at all because it never moves in any particular instant.

*The Stadium*

Zeno's fourth argument, the Stadium, is his most controversial. It is not only the resolution of the problem which is undecided, but also what it is that Zeno actually intended the argument to mean. Because of this, various sources have given two discrepant problems relating to Zeno's Stadium argument and corresponding resolutions. Between these two interpretations, however, it will be shown that the first interpretation has a much stronger case for being accurate in regards to Zeno.

The first deals with the apparent "jump" in two equal bodies moving in directions opposite to a third static body of also equal size. For simplicity a diagram will be used:

$A^1A^2A^3$←Moving Body A
$C^1C^2C^3$ -- "Stadium"
$B^1B^2B^3$→Moving Body B

This is the stadium at one instant. If time and movement occur in discrete units, in this case the length it takes one sub-letter of the body to move one letter length, this is what the stadium will look like in the next instant should bodies A and B each move one space in opposite directions:

$A^1A^2A^3$
$\quad C^1C^2C^3$
$\qquad B^1B^2B^3$

Zeno's problem is that in the first instant $A^3$ is aligned with $B^3$ and $C^3$. However in the next instant $A^3$ is aligned with $C^2$ and $B^1$. When was $A^3$ aligned with $B^2$? How did this "double-jump" occur? Zeno argues that because of the time is composed of discrete units for this argument they never meet, though logically and in sense experience they do. This is the more popular interpretation of the argument, as it is the one presented in

Aristotle. (Aristotle, 239b33-240a16)

The second interpretation is much simpler and deals with a finite maximum velocity. Zeno says that if motion is discrete at a base level, then it must also be discrete as the values of such velocities become greater and greater. Simply put, if there is a smallest particle, a smallest time interval, a smallest measurement, then likewise there must also be a largest of each of those corresponding constructs. Let us assume that there is some maximum speed. If A is looking at two bodies, B and C, approaching each other from the left and right, and the two bodies are approaching each other at the maximum speed, then they are approaching each other relative to A at twice the maximum speed. This clearly leads to a contradiction, and thus there can be no finite maximum speed. However, this seems to be more of an application of the first interpretation. Most likely this is merely nothing more than that, and though it is logically sound, one should not confuse it with the genuine Zenonian argument that is presented in Aristotle.

## THE PARADOX

Now Zeno's paradox emerges: if motion is logically inconsistent, if motion is continuous and infinitely divisible, but it is also logically inconsistent if motion is finitely divisible and discrete, there can be no motion because for motion to exist there must be a logical paradox! Indeed this has been food for thought for mathematicians, philosophers, and physicists for over two millennia.

## Logic

First and foremost, Zeno was a philosopher and logician.[3] It is very easy, however, to view him in the fallacious way in which many philosophers and scientists alike tend to: to consider him a scientist and use scientific or mathematical theory to disprove his arguments (which are rather easy with modern theory) and cast them aside for no further consideration. When one does this, however, one misses the entire idea which drove Zeno's formation of the arguments. It is beneficial and somewhat necessary then that before a scientific exploration of Zeno's arguments that his philosophy and logic be examined – in the area in which he participated – so that Zeno may be confronted "on his own turf," so to speak.

### REDUCTIO AD ABSURDUM

The style of argumentation which Zeno used in his attempt to disprove the existence of motion is indeed a powerful one, formally known as a *reductio ad absurdum*. In formal logic, a *reductio ad absurdum* is defined as follows: if some statement, proposition, premise, etc., which will be called P, is true, then Q is also true. However, it is logically impossible, given P, for Q to be true. Thus, P is also untrue. To present a formal definition using the mathematical notation, the *reduction* is as follows: $((p \supset q) \& (p \supset \sim q)) \supset \sim p$. Literally interpreted, this means if both p then q and p then not q, then

[3] Though Zeno lived before any formal definitions of logic were formed, one looking back may surely accept that had such foundations been set then Zeno would have called himself a logician, and on this basis he should be considered one regardless of the legitimacy of his arguments.

not p.  As simple a formula as this is ostensibly, it has proved to be an invaluable tool for mathematicians and logicians through the centuries.  The first known use of the reductio was by Pythagoras in the 500's BC, when he proved that the diagonals of a square cannot be measured with the sides.  By first assuming that perhaps the opposite was true, that the diagonals could be measured by the sides, and then showing how such an assumption would lead to a logical contradiction, he concluded that his original hypothesis was verified.  This is also the first example of one of the mathematician's greatest weapons, the indirect proof.  To see how vital this method of reasoning has been to mathematics, one only needs to refer to the copious mathematical theorems that have been verified through this method and are thus accepted as true.  One of the most notable examples is the famed proof of the irrationality of the square root of two, first published by Euclid in his *Elements*.  It is first assumed that the square root of two may be expressed as the ratio of two numbers (the nominal property of a rational number) that, when squared, equal two.  However, one must accept that the denominator is both rationally indivisible and may also be divided by two, which is clearly contradictory.  Ironically, it was considered Pythagoras's greatest downfall as a mathematician that he refused to accept the existence of irrational numbers  He held firmly to the ideal that the universe was governed by whole and rational numbers, and to even propose that numbers that could not be presented as the ratio of two whole numbers was preposterous and blasphemous; the Pythagorean student who first presented to Pythagoras that there are some numbers that cannot be so written was put to death by Pythagoras's order.  Please refer to Appendix I for a thorough proof or the irrationality of the square root of two.

PER IMPOSSIBLE

Sometimes Zeno is given the credit as being the author of reductio reasoning; however, as noted above, it is actually Pythagoras who was the true father. What Zeno did utilize first was a method of *reductio* argumentation known as *per impossible* reasoning. Though it may be difficult to separate this from the general *reductio* argumentation, *per impossible* reasoning occurs when one attempts to establish a conclusion with premises that are inherently impossible. If the premises are contradictory, then one may imply any conclusion they wish. This statement may be stated symbolically as $P \cdot \sim P \supset Q$, where Q represents any statement at all. Zeno's argument was based upon this style of reasoning: it is impossible for motion to exist, because to assume motion exists does means that one may cross an infinite distance in a finite time (as he shows by his Dichotomy and Achilles arguments), or that something may be both where it is and where it is not simultaneously (as shown by the Arrow).

## ARISTOTLE

As mentioned previously, Zeno's arguments were left unchallenged until Aristotle. A question has been ignored but must now be presented: is Zeno logically consistent in his arguments? Is there, if perhaps not a scientific ground, a logical ground on which to accept Zeno's paradox? It is necessary to turn to the original source from which all knowledge of Zeno's arguments comes: Aristotle's works.

Aristotle mentions Zeno in several places throughout the massive body of his work, and frequently uses him as an example. Zeno's paradoxes are explicitly given in Book VI of Aristotle's *Physics*. It is not necessary to relate Aristotle's explanation of Zeno's arguments here; they are expounded in the previous section. There are, however, some points that are pervasive of Zeno's four arguments to consider. Zeno's Dichotomy and Achilles are based upon the same principle. Is an infinite distance required to be traveled as a result of Zeno's division? By using Zeno's analogy the answer is, in a word: no. What Zeno has committed here is a logical fallacy that has been a danger for at least as long as the time of Zeno, and, indeed, is a danger necessarily as old as deductive reasoning itself, a fallacy of equivocation. An equivocation is committed when one shifts the meaning of a term in an argument, and thus an unproven conclusion is invalidly reached. What Zeno equivocates is infinite and infinitesimal, two terms that are based on the same principle (namely unending measure) but are quite different. This fallacy may be expressed no better than by Aristotle himself:

> *Hence Zeno's argument makes a false assumption in asserting that it is impossible for a thing to pass over or severally to come in contact with infinite things in a finite time. For there are two ways in which length and time and generally anything continuous are called infinite: they are called so either in respect of divisibility or in respect of their extremities.* (233a22-26)

Think of the fallacy in the following sense: though one may divide a ruler into an indefinitely great number of points, such divisions do not make the ruler any longer that it was originally. Apparently, such a discernment of the two terms has been exceedingly difficult for many minds throughout Western history. Aristotle also takes the assumption of dividing length into infinite points to make it infinitely long and applies it to time: one may take time and divide it infinitely, and in doing so then comes into contact with infinity of time. With infinity of time at one's disposal, one may then easily traverse an infinite distance. Thus is driven the first of the two nails into Zeno's logical "coffin." But there are counterarguments to be made in Zeno's defense, which will be given later.

In regards to Zeno's Arrow, there are several objections to be considered. Firstly, Aristotle makes a claim far ahead of his era; the Arrow is wrong because Zeno fails to account for time. Aristotle says in Book V of his *Physics* that "everything that is in motion is in motion in time and changes from something to something" (239a23-24). The claim implies that no motion occurs outside the consideration of time, and that motion must necessarily be measured in regards to time. An expounding of this may be found in section V. To retain pertinence to Aristotle however, it must suffice for now to declare such a relation only in the general sense. Since motion occurs in time, even to attempt to define motion outside the consideration of some time interval is absurd. Conversely, the absence of motion, or what would be termed "rest" in the context of the Arrow argument (if the arrow isn't in motion it is at rest), must also be measured over an interval of time; one cannot tell if something is at rest when one cannot tell if something is in motion. Take a simple analogy: how can one decide whether or not some person is male when one does not know whether or not that person is female? The presence of motion or rest falls upon the same reasoning. Therefore, when Zeno tries to define motion in some discrete, particular "now" (as termed by Aristotle, synonymous with an instant) in which there is no period of time to consider, one cannot conclude that an object is in motion or at rest; "for at a now it is not possible for anything to be either in motion or at rest (239a37-239b1)." If these aren't sufficient for the reader, Aristotle also attacks the proposition of a universe comprised of indivisible units. He claims that no magnitude may be composed of indivisible units, much less time (Ch. 9). In his *On Indivisible Lines*, Aristotle discusses the existence of a smallest discrete magnitude, beginning by stating the opposing view that one does exist. Using Zeno's Dichotomy as the evidence, the proponents of the theory believe that because one cannot encounter an infinite number of things in a finite time, there must be some smallest magnitude, or as the title of the work suggests an indivisible line, to eventually cross and dispel the seemingly resultant logical contradiction.  Aristotle corrects them, stating that in a certain sense infinite and finite occur simultaneously because though the number of points are infinite, the distance is finite.

Aristotle makes another brilliant point, millennia ahead of his time, by stating that there are also differences in infinity; that though some things are infinite in the sense of one by one counting, some things are rather potentially infinite in the sense that one may continue adding possibilities so to speak forever, and thus it is impossible to even begin "counting" as is possible in the first case.  What makes this reply so ahead of its time is that this argument is identical to the foundation of modern day set theory, which is discussed in section V.  While in the vein of modern science, what one also finds here is, in theory, an ancient argument disputing the existence of supertasks.  The possibility of supertasks has emerged only in the last few decades, as a by-product of quantum theory. Further explanation of them may be found in section VI.

Zeno's Stadium was seemingly the easiest argument for Aristotle to refute, and unlike the other arguments this one is almost unanimously considered logically erroneous from the get-go.  What Zeno assumes for some reason is that every sub-letter of the bodies in the example in section IV must meet with each subsequent sub-letter at each discrete instant.  Zeno fails to notice that making this argument also assumes continuous motion, even though for this argument Zeno is assuming that motion is discontinuous and discrete.  Therefore the sub-letters of the bodies can logically make the "double-jump" between one discreet instant and the next (Grunbaum, 1967).

A few points should also be made on Aristotle's comments on Zeno's paradox as in general. Aristotle mentions in his *Sophistical Refutations* the difference between showing a false conclusion and accurately revealing the root of that conclusion:

> *There is nothing to prevent the same argument from having a number of flaws; but it is not the exposition of any flaw that constitutes a solution; for it is possible for a man to prove that a false conclusion has been deduced, but not to prove on what it depends* (179b17-20)

What is the implication to modern students of logic? What Aristotle says, in simpler terms, is that Zeno has committed one of the basic fallacies of logic in his arguments (it should be noted the very system of Western logic has an Aristotelian foundation), the accident. An accident occurs when the premises of an argument are logically irrelevant to the conclusion. For example, one concludes that because all apples have a core, and because apples and oranges are both fruits, oranges must also have a core. Here the flaw in reasoning is clear; the commission of such a fallacy is synonymous with the old cliché of "comparing apples to oranges." Similarly, what Aristotle means in the aforementioned passage is that simply because one demonstrates the logical error of an argument, as in Zeno's case where he shows the contradiction inherent in his arguments, he doesn't necessarily show the reason for the contradiction in the arguments – simply put there is no evidence that the contradiction emanates from the nonexistence of motion.

Aristotle offers final words of warning for those who may be perturbed by these arguments. In *On Indivisible Lines* Aristotle rebukes those who follow Zeno's line of reasoning because of their inability to logically satisfy a refutation of it: "It is surely absurd that, because you are unable to solve Zeno's argument, you should make yourselves slaves of your inability, and should commit yourselves to still greater errors, in the endeavor to support your incompetence" (969b4-6). He repeats his warning a few lines later, writing "Moreover, it would be absurd for people to be led astray by Zeno's arguments and to be persuaded—because they cannot refute it—to invent indivisible lines" (969b17-18). A rebuke as important to note in modern times as it was for Aristotle's contemporaries

ZENO SURVIVES

The first attack on Zeno was indeed crippling, and those who follow the Aristotelian mode of logic – which is most any sensible person – would surely agree that Aristotle retains logical consistency throughout each of his critiques of Zeno' paradoxes. It is indeed difficult to consider the paradox of motion in a spirit of gravity in light of Aristotle's refutations. However, logical consistency is not alone sufficient to produce a powerful argument. As any novice logician knows, a sound argument requires a set of premises having two characteristics: a conclusion that cannot be false if all the premises are true and the premises themselves are all true. As has been made transparent over the centuries, by way of science and mathematics, Aristotle failed to deliver in terms of the latter characteristic on his arguments. So, even Aristotle's massive intellect could not snuff out Zeno for all time. Nonetheless, one wonders why Zeno's paradox has continued to survive in the arena of conjecture despite such crippling blows inflicted upon it.

Two answers are appropriate: the second answer one may conjecture to be likely resultant of the first. The first reason is one based on psychological grounds. The desire to question is part of human nature; if it weren't then science and the empirical method would have never sprung into being. To question and wonder is indeed a very fundamental part of what makes us human. One cannot help but be reminded of the famous maxim of Socrates recorded in Plato's *Apology*; at Socrates trial he exclaimed to his accusers that "the unexamined life is not worth living" (Plato, Ap. 38a). It is not overly difficult to grasp as well that the more commonly-held and perceptually fundamental a belief is, the more intriguing any doubt, no matter how flawed or incomplete it is, against that common notion would be.

To remain fair to logic, however, those who are skeptical of Aristotle's critiques are afforded some ground. Aristotle offers solutions to all of Zeno's arguments, but two of these solutions[4] share a common foundation - one which is shaky enough to have allowed a continued faith in the seriousness of the paradox.

As a general response to the Dichotomy and Achilles arguments, Aristotle claims that magnitude and time are dependant upon one another. If Zeno believes that an infinite distance must be crossed to reach any point, then Aristotle claims that time may be stretched in the same way as magnitude so that there is an infinite amount of time in order to reach the point, making the "infinite" distance needed to travel possible. For the Arrow, Aristotle writes that one does not reach Zeno's conclusion regarding the motionless arrow if one does not begin with his assumption that time is not composed of point-like "nows," but rather discrete units; "if this assumption is not granted, the conclusion will not follow" (239b31-32). One must ask himself, has Aristotle actually defeated Zeno's claims, or has he merely tried to find a tidy way to "patch things up?"

[4] One must keep in mind that the Dichotomy and Achilles are based upon the same argument, and the Stadium argument is inherently flawed. Essentially, only two arguments need to be considered in order to critique the paradox.

Aristotle himself claims that a mere declaration of the falsehood of a statement is not sufficient disproof, but does he subscribe to his own philosophy with these two critiques? Aristotle makes an error here in making assumptions which he is not warranted to make. To say of the Arrow that if one does not make the assumption the conclusion does not follow has no detrimental effect on the argument, it being merely a tautology. Of course if one does not make the assumption then the conclusion will not be reached! Zeno does not argue this, but rather what the consequences of making such an assumption are. So then Aristotle's only real argument here is that Zeno is wrong because it is certainly true that time isn't composed of discrete instants. The problem though is that Aristotle himself offers no proof![5] Whether or not this conclusion seems more reasonable or not is of little concern to one who requires reasonable evidence for any claim; with no evidence, either empirical or by common sense derivable from pure thought, Aristotle is merely pitting his own personal opinions against Zeno's, and whether or not Aristotle's sounds more reasonable or not is immaterial, at least if one is attempting an internal critique of Zeno.

A similar mistake is made in Aristotle's "time and magnitude argument." Aristotle's construction of this argument rests upon assuming that one may not determine locomotion or time without the consideration of the other, an assumption which Zeno never made. As far as Zeno was concerned, and as far as Greek mathematics and science had discovered, time had nothing to do with locomotion. Not only is the claim that time and magnitude are necessarily dependant on one another completely without foundation, at least not at the time of Aristotle's writing, he once again fails to recognize that Zeno

[5] We might, however, upon study of Aristotle provide some of our own. For example, time is metaphysically conceived in terms of distance (i.e. a line in space). Just as a line segment isn't composed of points, so time isn't of instants. (from J. Garrett)

makes no such claim himself, and during that time Zeno had every right to not assume that motion and time are dependant on one another. Zeno could also just as easily rewrite his argument to say that one must cross an infinite amount of time in order to reach the next point, in which case his argument stands unchallenged again (de Sousa). What one finds is that in essence Aristotle has claimed that Zeno is wrong because his arguments (excluding the Stadium) are logically fallacious under Aristotle's conception of physics, which was largely a compendium of the common sense view of the physical world in ancient Greece, a view which science has determined to be, though certainly the most well-thought and sophisticated of its time, largely mistaken.

These are the attacks which have been made upon Aristotle regarding the paradox, veiled by skepticism yet with a secret resolve to perpetuate Zeno of Elea as a serious figure in philosophy. Throughout the past 2,300 years these attacks have been unanswerable strictly within the scope of philosophy. There is one argument that has been constantly overlooked though - which has been purposely left unmentioned until now. Aristotle also claims, as is mentioned in the previous heading, that there are two kinds of infinities: one in relation to division and one in relation to extremities. This argument is unchallengeable because it points out a basic logical fallacy (as noted above an equivocation). This argument may defeat Zeno's infinitely divisible, continuous motion arguments single-handedly, and most likely would have in ancient times had the Western world possessed an understanding of infinity as progressive as Aristotle's. For this reason the argument was overlooked, because it was not until the time of Leibniz and Newton that this distinction between infinite and infinitesimal took firm hold in the scientific community.

If one uses only Aristotle's refutations as a basis for physics, and commits oneself to the presented problems in Aristotle's refutations, one will be led to logically conclude that motion occurs continuously. This is exactly what Isaac Newton concludes, and in the process constructs a physical system in which time and space are absolute, and though measurements of one are made in relation to the other mathematically they were assumed to exist independently of one another. This classical mechanics was the next step in the evolution of space, time, and motion, and is discussed in the next section.

## Science

After the consideration of Aristotle, Zeno lay dormant. Though being mentioned in the cumulative accounts of Western philosophy, any consideration of his paradox has escaped public record, and to consider the absence of such a thing as motion in physical existence would most likely have been tried as heresy under the reign of Roman Catholicism. Rather, philosophy turned its head away from such "outrageous" speculation after the fall of the Roman Empire and as both Christianity reigned and the source of power spread from Rome to all of Western Europe philosophy began to worry itself with new topics - namely arguments for the existence of God and discourses on Christian theology. Indeed, some of the most profound philosophy of the Western world came from these Christian philosophers – namely Augustine of Hippo and Thomas Aquinas – but never did philosophy enjoy the freedom and governmental support granted to it in Ancient Greece and Rome, that is, post-Socratic Ancient Greece and Rome.

### CALCULUS & CLASSICAL MECHANICS

Though, as far as this author and many others are concerned, Zeno's paradox was exposed as logically fallacious, the first great mathematical breakthrough in proving why Zeno was wrong coincided with one of the greatest discoveries of mathematics, the Calculus. As any amateur historian of mathematics knows, the discovery of this branch of mathematics is among the most storied of all history, including the romantic imagery of Newton writing his *Principia* in the cold attic of a cottage during the winter of 1665 (Beckmann, 1971) and the ensuing debates between Newton and Leibniz as to who was the true discoverer of the Calculus; however it is sufficient to strictly cover the pertinent mathematical theorems, as enough authors have written such biographies with more beautiful literature than is necessary for this study.

The first of these mathematical answers to Zeno's paradox lies within limit theory. Limit theory is the first thing that is taught in an introductory calculus class; the whole of calculus rests upon the understanding of limit theory's simply-stated yet elegant basis: a mathematical function can be made to be as close to some value as desired by making the variable in the function sufficiently close to a certain value. Symbolically written, one may say $\lim_{x \to c} f(x) = L$, where x is the variable in the function, c is the answer for the function desired, and L is the value of x that brings the output of the function as close to c as is mathematically possible. For examples illustrating the limit function, please refer to Appendix II. Zeno's first two paradoxes are solved by a specific type of limit known as the infinite series. An infinite series is an infinitely large group of numbers that becomes increasingly close to some number, though no number in the series ever ever converges upon or becomes equal to that number. The first infinite series to be considered in the *Principia* was a mathematical representation of Zeno's paradox, which Newton was as well aware of as any of the other scientists and mathematicians of his day. The series is a group of numbers beginning with one half, with each subsequent term in the series being one half of the previous term added to the sum ad infinitum. One has a series consisting of $1/2 + 1/4 + 1/8 \ldots 1/\infty$. Zeno's incorrect assumption was that the sum of this series was infinity. The conscientious reader may follow that this sum is actually equal to one; however this is only partially right. Because the series is never ending, the sum can never actually achieve unity, or become equal to one. Rather, this problem is better stated "the limit of this series as x (representing the number of terms in this series) approaches ∞ is 1." With this statement the series may then be made as arbitrarily close to one as desired in practical application, but is only truly equal to one when there are an infinite number of terms present. With this example, mathematics proved that an infinite series of numbers can actually have a finite sum (in the case of the example above, the sum being one), and that Zeno was at least half wrong – the Dichotomy and Achilles arguments were based upon faulty reasoning and it could be proven with numbers rather than words.

As noted in the previous section, during the time of Newton in Europe one who was to consider whether the universe was continuous or discontinuous in light of Aristotle's and Zeno's arguments and the weaknesses thereof would gravitate[6] toward the belief of discrete motion, a universe having absolute space and time. As Newton based his physics upon this proposition, he made a mathematical attempt to answer the last remaining argument Zeno presented 2,000 years ago, the Arrow. One of the elementary properties of classical physics is velocity, the rate of change and direction of a body's position over time. In Newton's genius, he was able to incorporate the rules of calculus to describe physical properties and processes. The velocity of an object can be calculated by taking that body's position, which can be defined by a mathematical function, and finding its derivative, a tool of calculus. By taking this derivative, which is itself a function, one may determine the velocity of that object at any instant. This type of calculation is known as differential calculus (a differential is defined as an infinitesimal

[6] Pun intended

change in a value, for the purpose of determining speed that variable is time), which along with integral calculus comprise the mathematical field's two branches. The speed of an object determined by this calculation is known as instantaneous or differential speed.

By the 1700's, three major systems in the progression of man's knowledge of physical reality (Aristotelian physics, classical mechanics, and calculus, though the first is incompatable with the second two and had by this time mostly been shown mistaken) had arisen as the result of three of the West's greatest scientific minds, and systems concurrently made gallant attempts to slay the troubling paradox that was constructed by Zeno of Elea. Though Aristotle's attempt, which was itself plagued by small discrepancies, was not sufficient to quiet those who may advance a theory of nonexistent motion, surely this new science and mathematics that could internally answer all known physical questions in existence, and prove it with numbers, would deal the killing blow. However, once again Zeno's paradox escaped the clutches of certain perishment, though scathed, with enough wiggle room to give doubt to this new physics and mathematics. This doubt was nestled in the nature of the limit theorem. A skeptic of calculus may hold a philosophical view that it is an invention born out of man's own incapacity to deal with the infinite. What does the necessity of the limit theory, when a variable is approaching infinity, imply? Because humans cannot actually solve a problem that requires calculations of infinitely long numbers, a method must be devised that approximates such values to an arbitrarily accurate degree. This method of approximation, though able to gain accuracy excellent enough to build bridges and gauge the speed of trains, is not exact, and anything less than exact is not enough for a logician or philosopher who holds a certain view, especially when one does not wish to discard such a view anyway. For a similar reason, skeptics did not believe that differential speed was exact enough to disprove the logic of Zeno's Arrow argument. One must consider velocity conceptually to respect this skepticism. Velocity is by definition a change in position over time, or dx/dt, where x is the position and t is time. Instantaneous velocity is not the change in velocity when t = 0 as some think, because to make t = 0 would lead to no solution, as any ratio with zero as the denominator is taken to "blow up" to infinity. Rather, instantaneous velocity is the ratio of the change in position to time as t approaches zero, a limit. Just as one cannot obtain an exact answer with infinitely long numbers, so one cannot obtain an exact answer for an infinitesimal interval of time, t = 0, and is thusly required to approximate.

The question of the validity of these approximations was not solely made by proponents of Zeno's paradox; rather, this question was trifling compared to the more practical ramifications of these approximations. Though the approximations of infinitely long numbers are adequate when dealing with small-scale questions, such as those in engineering and "real-world" applications, what happens when the need for approximations of numbers closer to infinitude in both the macro and micro sense occur, say when determining astronomical problems such as planetary motion or the movement of particles within an atom? The answer is that calculus simply cannot account for motion on grand and minute scales, and the discovery of the physical and mathematical truths that do is the big story in science from the 1700's to the present.

## SET THEORY

The next advance in cracking Zeno's logic with mathematics came in the 1870's.

The end of the Nineteenth Century was a pivotal time in the history of science. Experiments and discoveries that questioned the validity of the most basic of physical and mathematical truths were being published by some of the most brilliant minds in human history; it was these minds that sparked the doubt and zeal for innovation in the scientific youths of this age that would lead to the discoveries that have caused an upheaval of the whole domain of scientific understanding. Before such understandings could manifest, however, mathematics that could accurately describe such theories would be required. One of these early mathematical discoveries was part of what has become set theory, a widely-studied branch of number theory. Number theory is the study of the relationships between numbers: what is called "pure mathematics" because it is the study of numbers as entities in and of themselves and not merely as symbols for practical application. Number theory deals with the same questions regarding infinity that were raised first by Zeno of Elea in his paradox

In 1874, the German mathematician Georg Cantor published one of the seminal papers of modern mathematics "On a property of the set of real algebraic numbers," a paper from which the foundations of modern set theory emerged. Set theory is a branch of number theory that describes the nature of collections of objects without mass or energy (that is to say, abstract mathematical objects), and the relationships between these objects or elements (the technical name for the objects that comprise a set). Set theory has had vast implications for the modern understanding of infinity, and the relationships between infinite sets of objects.

Set theory is best introduced by explaining it in terms of finite sets. A finite set is, as common sense would imply, a set that has a finite number of elements. A set with no elements, called the empty set, is also classified as a finite set and denoted by the Greek letter Φ (phi). Set theory is not particularly interested in lone sets, however, but rather the relationships between sets. For example, {1, 2, 3, 4, 5} is a finite set with five elements. Likewise, {boy, apple, carrot, car, tree} is a finite set with five elements. Both of these sets share cardinality, which is defined as the characteristic which a set shares with any equivalent set. For any set with a finite number of elements, the cardinal is simply the number of elements. The two sets in the above example share a cardinal of five. When considering infinite set, it is deceptively simple to say that all infinite sets are infinite, since they all have an infinite number of elements, and therefore share a cardinal of infinity. Fundamental to set theory, however, is the principle that not all things that are infinite are equally thus, therefore some things are "more" infinite than others.

Infinite sets are considered to be equivalent if each element of the set could theoretically be put in a one-to-one ratio with one another. For example, it is a basic axiom of mathematics that there is an infinite number of both odd integers, denoted by the letter O, and prime numbers, denoted by the letter P (though this set's infinitude is debated more than the former's). However, it can be shown that they are equal because if one were to write out the entire list of each set, each element of one set would be paired up with an element of the other set. For these two sets, the first positive odd integer, 1, can be paired with the first prime number, 2. The next odd integer, 3, can be paired with the next prime number, which is also 3. These pairs between the elements of these two sets are unique, and this pair method may be continued ad infinitum, theoretically.

Common sense would assert that among these numbers, the set of rational numbers would be greater than the sets of odd and even integers, primes, and rational numbers, since all of these sets are arithmetic subsets of rational numbers. In other words, all of the sets mentioned above are included in the set of rational numbers, but not every rational number – in fact most rational numbers – is a part of one of these other sets.

Yet before jumping to this conclusion based upon common sense, it is necessary to look at what Cantor says about equivalence, expressed in Cantor's Theorem. Each set may be divided into a certain number of subsets, consisting of each element and any combination of each element. The number of these combinations is called the power of the set. For the set {1}, this power set consists of {Φ, {1}}. For the set {2, 3, 5 }, the power set is {Φ, {2}, {3}, {5}, {2, 3}, {2, 5}, {3, 5}, {2, 3, 5}}. From these two examples, one may draw a general formula for the power of a set. The power of a finite set is equal to $2^n$, where n is equal to the number of elements in a set. The powers of the two example sets are 4 and 8, respectively. From this formula Cantor's Theorem is drawn: the power set of any set S is larger than S. Though this is easy to see for finite sets, it requires some abstraction for an infinite set. Any two infinite sets are equal if they share cardinality, if each element of the sets may be paired in a one-to-one ration. So, looking at the set of rational numbers and the other infinite sets that are seemingly smaller than it, one finds that actually they are all equal!

From Cantor's Theorem it would seem to hold that any infinite set is the same. However, upon examination one may find that this is not the case. The set of real numbers is in fact larger than any of the other categorical sets (integers, primes, rational numbers, etc.). This is so because the elements of the set of real numbers cannot be placed in an ordered pair with the elements of any of the other sets. In fact, should one take on the task of counting the real numbers, one would be quite daunted in even trying to begin – the first element of the set of real numbers would have an infinite number of decimals. For this reason one cannot even count between one real number and the next, save the ones that have a finite number of digits such as the elements of the smaller sets mentioned above, which constitute only a relatively infinitesimal portion of the entire set of real numbers.

Though the sets of integers, rational numbers, and company are countably infinite, the set of real numbers is uncountably infinite. In this comes the error that led to the formation of the Dichotomy and Achilles arguments: a division of length that is uncountably infinite, such as the division Zeno makes when he separates the distance into halves ad infinitum, is equivalent to infinite length. Yet, all that these arguments say in answer to the logic of Set Theory is that the number of possible divisions within any interval, such as between 0 and 1, is uncountably infinite. Nonetheless, the interval itself remains finite, and so the arrow hits the target and Achilles overtakes the tortoise, because these arguments lie in Zeno's confusion of countable and uncountable infinities.

## RELATIVITY, MINKOWSKIAN SPACE, HEISENBURG UNCERTAINTY PRINCIPLE

The story and theory of relativity, special and general, is much too deep and convoluted to recount without the addition of a book-length study. Thus, it must suffice to resort to brevity for the purpose of advancing this study toward the applied material. Einstein, Minkowski, and Heisenberg all stand as giants in the realm of physics, however, and to discredit them as mere workhorses for the purpose of this paper is quite pretentious. It is highly suggested that one engage in study of each of these brilliant pioneers, as they stand in the same Hall of Greats as Newton, Leibniz, and Cantor.

Until the implementation of general relativity, there was no satisfactory solution to the Arrow argument. The classical image of space and time, which Zeno clearly argued against, assumed that time is composed of a series of instants. In relativity, however, space and time are not independent, and the basic way in which space and time fit together corresponds with Zeno's argument. In the briefest way of putting it, objects in relative motion do not exist on the same plane of simultaneity, and thus not only do objects in motion appear different to nonmoving objects (the world), but the nonmoving objects also appear different to the moving perceiver. (Mathpages, 2006)

In light of the Heisenberg Uncertainty Principle, the evidence is further solidified. This principle states that one may not precisely know both the position and movement of a moving object simultaneously. Position and movement are now two constructs which are all but undeniably incompatible with one another in terms of simultaneous measurement. Thus, the classical "sum of instants" model which existed until the twentieth century has been shown as a theory that is invalid for understanding motion on "any and every" scale.

Also, modern non-quantum physics uses a universe in its calculations known as Minkowskian (named after its discoverer, Hermann Minkowski) space, which consists of four dimensions—three space and one time. This division of space into four dimensions, which is usually shown graphically in two dimensions of space and time, highlights the dependence of space and time upon one another. Some modern problems and theories associated with this problem are introduced under the Motion and Time heading of Section VI**.** For the second interpretation of Zeno's Stadium argument, Minkowski also offered the concept of $G_\infty$, which essentially explains that there may be no upper limit on velocity (Mathpages, 2006). Here one also has an alternative to the idea of a limited maximum velocity, an answer stating rather bluntly that there is none.

In 1927, amid the debate between relativity, which had only come into existence in 1905 with the publication of Einstein's legendary paper on special relativity, and the startling discoveries that would lay the framework for quantum theory came a discovery that would rock the foundations of the world. This discovery was made by Werner Heisenberg, and the concept which was the fruit of this discovery now bears his name: the Heisenberg Uncertainty Principle.

This principle states, in the terms most accessible for non-physicists, that the position and momentum of any object, from a quark to a planet, may not be both known precisely at any instant. As the accuracy of one of these measures is increased, it is a mathematical conclusion that the accuracy of the other measurement will decrease. Interestingly, this principle is partially supported because it has incidental ties to a physical value: the product of the two measures of uncertainty ($\Delta x \Delta p$) equals a number that is some multiple of Plank's constant, a number which manifests itself in a number of equations across several scientific disciplines.

Theoretically, this principle implies that the world as seen by classical physicists is wrong. It was a common belief that one could not only know both the exact position and momentum of a particle at any instant, but that one could then determine the future of that particle for an arbitrarily extended time, on to infinity. This principle states that just the opposite holds, that the position and momentum of any object may only be known to a certain degree at any instant, and so its fate is partially up to what appears to us as chance. This also implies an even stronger relation of motion and time: because one cannot measure the position and momentum of an object at any single instant it is necessary to measure on an interval.

A single, succinct paper which explains how physics has traded precision for continuity in its measurements of a body's position, and thus has resolved all of Zeno's paradoxes, save the Stadium argument, was written by Peter Lynds (2002). A review of it may be helpful in understanding modern physics' solutions to three of Zeno's arguments.

## Application

Although it is a sad reality for those who wish to overturn apparently unshakable ideas, mathematical and physical systems have been incorporated which may satisfy even the most skeptical freethinker, assuming he or she submits his or her decisions to the most reasonable evidence available.  This is notwithstanding the commitment of oneself to a principle in the philosophy of science that is in acquiescence with a common standard in Empirical philosophy: that one should tentatively accept as truth that which has the greatest probability of being right, given the available evidence.  So the question which follows is: what use does Zeno's paradox serve in a world torn between principles of relativity, quantum physics, and other theories, all hoping for an all-inclusive synthesis between them?

Even if the paradoxes serve no practical purpose for modern academia, they have served a phenomenal role in the history of human thought.  The reality of this is manifest throughout this work, as it is hopefully clear to the reader that these paradoxes have called for the greatest thinkers of Western civilization to ponder them for the purpose, if nothing else, of solving them for personal satisfaction.  As a product of this desire to solve them, the most crucial of scientific and mathematical theories have come into being.  Though it is exaggerated to say that these paradigms of science are direct results of thinkers attempting to provide answers to Zeno, the fact that Zeno has been mentioned or discussed by every one of these “fathers” of science and mathematics makes clear that his paradoxes at least glimmered in the minds of Aristotle, Newton, Leibniz, Cantor, Einstein, and company.

An even greater contribution has been the dialectic. Though the dialectic has been more commonly attributed as an invention of Socrates (as the dialectic is also named the “Socratic Method”), Aristotle and philosophers succeeding him have rightly attributed the dialectic to an innovation of Zeno of Elea, though this fact continues to elude the knowledge of the layman. For this reason alone Zeno of Elea deserves a spot in the pantheon of intellectual greats. Yet, beyond this his paradox continues to fuel theoretical innovations across the spectrum of science and mathematics (and philosophy), and two answers for why his paradox matters are offered to the vehement skeptic.

As a more superficial, though independently satisfactory, answer the paradox strengthens human understanding of the history of thought, and serves as a tool by which to strengthen one’s faculties of reason, as was initially stated in this work. Though another explanation of this benefit is unnecessary, it may be profitable to examine some of the more lighthearted work done by way of contemporary philosopher/mathematicians. These individuals have taken Zeno’s original arguments and modified them slightly so that the original errors within the arguments are eradicated. Though these “puzzles” have not led to any breakthroughs in human understanding of the universe or any deep mathematical truths, at least not as of yet, they are presently considered unsolved. The reader may take great enjoyment in contemplating these puzzles for his or herself, as the most illustrious of which are outlined in Appendix III.

Yet, more importantly, the paradoxes do serve vital roles in more than one academic field. They are not merely intellectual fodder to mull over for leisure, but rather are foundational or preliminary of very radical views of the universe and humanity. It is self-evident that motion and change exist. Yet in light of the discoveries of the past century, perhaps it is time to revive the Eleatic vision, and question the perception of continuous motion. It is doubtful that most people have ever questioned the nature of motion, but rather accept that what appears to the senses is reflective of reality. This notion is known to philosophers as naïve realism, and the rather unflattering name reflects the lack of intellectual rigor involved in holding this view. Even the colossus of philosophy Aristotle held a view similar (though not equivalent) to this, referred to as metaphysical/epistemological realism, which holds that the senses are capable of leading the conscious being to a true understanding of reality.

Zeno and the other Eleatics, however, were acutely cognizant of the inadequacy of the senses. This awareness is illustrated in Simplicus' account of Zeno's millet seed paradox, which though should not be confused with the four central to this study is nonetheless wonderful in making this point. Zeno asks Protagoras if a millet seed, or one ten-thousandth of a seed, makes a sound when if falls. When Protagoras answers that it doesn't, Zeno then asks if a bushel of millet seeds makes a sound when it falls. As would be expected, Protagoras answers that it does, and Zeno makes his point from Protagoras' answers. If a bushel of seeds makes a sound, then some ratio of that, such as a single seed or one ten-thousandth of a seed, likewise must make a sound as well – yet humans cannot detect it. Zeno makes it clear that the senses are capable of fooling us, and are not to be trusted as the "perfect receptors of form" that many people were led by Aristotelian, and other, passages to believe. (Barnes, 1996)

Perhaps after reading the evidence provided, and studying these topics more deeply, the illusion that the reader reasonably knows how the universe operates is likely to be shaken, and the reader may scratch his or her head with the same confusion that modern scientists now imply in their research papers and books. So the challenge to the reader is this: read on and prepare to shake the "common sense" conceptions of motion, in the process opening up a door to a truer understanding of reality, compliments of derivative Eleatic philosophy.

## PHILOSOPHY

### *The Philosophy of Weird Science*

The tool which allows application of this most peculiar set of ancient arguments in "ultramodern" scientific theory is a bridge unlike any other ever constructed by man, the science of quantum mechanics. Before exploration of quantum theory (which may be found under the next heading) and what reasonable evidence it may provide, it is imperative that science be separated from science fiction, and good sense from the pull of mysticism.

Quantum mechanics is one of the most powerful tools given to the scientific mind. This is not a minority viewpoint among physicists. For example, E. H. Walker says of quantum mechanics, "It is safe to say now that nothing that we see in this world lies outside the breadth and scope of quantum mechanics…it describes everything in our world. It seems to have all the answers" (Walker, 2000, p. 68). The discoveries which are derived from quantum theory may easily topple all that has been previously held as true concerning the nature of man, the universe, and the particles and energy of which they are both comprised. However, the old cliché is in order: with great power comes great responsibility. It is unfortunately too simple to take a shred of the truth from observational evidence and distort it into one of the innumerable "new age" theories which have popped up in the past thirty years. All these theories seem to have the same foundation: take a tenet of quantum theory, such as the Bell experiments, the Heisenberg Uncertainty Principle, or wave-particle duality, or some part of special relativity, and begin to build a cathedral of faulty logic that allows for time travel, astral projections, and any other pseudo-supernatural phenomena which one may wish to insert. Jamie Whyte attacks such shysters of physics in his book *Crimes against Logic* (2005). In his section entitled "Weird Science," Whyte first takes aim at those who seek to promote their own outlandish claims, and then use the argument of scientific uncertainty to support their own claim. Clearly such an argument commits the fallacy of an irrelevant conclusion, whereby the premises are logically irrelevant to the conclusion. Whether or not science is good or bad, right or wrong, has no outcome on the validity of the claim being made. Whyte then turns his eyes upon quantum physics, saying that "not everyone who enjoys dabbling in conjecture wants to appear anti-science. For them there is always quantum physics" (Whyte, 2005, p. 40). He specifically calls into question the book in which the author states that because physics has now been forced to submit to the Uncertainty Principle, that science holds no truths and that there is no distinction between natural and supernatural phenomena. One must be careful; such statements are not only highly dangerous but are easily believable without proper physical knowledge. The author, Whyte comments disparagingly, not only fails to properly consider the scope of the Heisenberg Uncertainty Principle (which only pertains to simultaneously determining the position and movement of particles), but also begins to make claims about the merging of natural and supernatural and the complete destruction of science with no evidence directly relevant to such a claim. What one must be then is not only a good student of physics, but also one who adheres to scrupulous reasoning when presented with any claim, outrageous or otherwise. When considering a scientific claim, in particular theoretical physics, an individual must ask his or herself "How far does this claim stray from the observational evidence?" It is all too easy to take such shreds of truth from observational evidence and then begin to dabble into conjecture, as is seen in Whyte's example. Though each step in such an argument may follow logically, and the conclusion may seem to be correct, being logical must not be mistaken for being true: a mistake that is too commonly made. For example, one might claim that every hospital in the United States has begun electronically tagging newborn babies with a microscopic GPS tracker. Because of this the government now will be able to determine the exact location of any American anywhere in the world within a couple of generations. If they know where Americans are, they can determine numerous pieces of incriminating evidence; say, perhaps, the government may know if American youth under the legal drinking age are sneaking into bars by such tracking, and then may contact the police, who in turn hold stings at such bars that allow minors to enter. Granted, each of these premises may follow logically from the previous; however, the problem is that none of this is true! Though perhaps such faulty evidence is easy to see in this context, in science many books are written that utilize this same type of bogus evidence to support foolish propositions, and in doing so dupe masses of readers and fellow scientists alike into accepting and discussing claims which contain no validity. With this in mind, then, may the reader beware!

*Free Will*

A most novel philosophical topic concerning Zeno's paradox is that of free will. One such article, which has included Zeno's paradox as analogous to the question of free will (Mole, 2004), reasons that Zeno failed to realize finite and infinite are not mutually exclusive ideas. His Achilles argument failed to recognize that an infinite series can converge on a finite number, as has been shown previously. Seeing that because both the finite and infinite can coexist logically in this way, why may the lives of humankind not be both products of human will and the outcomes of uncaused causes simultaneously? Mole believes that though this may be counter intuitive to the way many think, the way these two ideas interact is just as logical as the way finite and infinite values interact in the Achilles argument. The mention of it here is purposely brief, however, and though no discussion of the argument's merit will be given it is encouraged that the readers find the original source and consider it for his/herself.

## PHYSICS

*Quantum Zeno Effect*

Most prominently, Zeno's paradox has found application in quantum mechanics. Quantum mechanics, as any physicist knows, is built upon finding the probabilities of possible observations at an instant in time, or a sequence of observations at different instants. What mathematical limit would one find should one seek to use an infinite number of measurements from continuous observation? George Sudarshan and Baidyanaith Misra first experimented on this question in 1977, and found a rather startling answer. Since, numerous quantum physicists also have researched this question and found what Sudarshan and Misra found: that continuous observation would cause no state change to occur, a problem known as the quantum Zeno paradox. Basically, this paradox shows mathematically that the probability of observing a quantum system in its initial state at any instant during continuous observation is one. What that means in practical terms is that math shows that no motion ever occurs in a system when it is observed constantly, which is how sentient beings seem to observe. Of course the non-existence of motion was a part of Zeno's philosophy. Others such as A. P. Balachandran and S. M. Roy (2002) have researched this question and concluded that constant observation mathematically ensures that change takes place in a system. What these findings suggest is that both answers are in fact logically consistent; the discrepancy lies in the setup of the experiment. In the experiments commonly done to search for the Zeno effect, there are two variables which are measured. The first variable, E, is made as a time-independent variable in the Schrödinger equation (which is used to determine the probability). The second variable, $E_s(t)$, is one which is time-dependant. One arrives at the Zeno paradox when looking to measure the variable E. Conversely, one arrives at what is now called the anti-Zeno paradox when looking to measure $E_s(t)$. Measuring the same probability for this variable with the same parameters as those used for the quantum Zeno paradox finds that the probably that a change will occurs is one, exactly opposite of the quantum Zeno paradox. It seems evident that this conclusion could only provide an even stronger piece of evidence for the necessity to measure space and time concurrently: that the Heisenberg Uncertainty Principle is mathematically and quantum-physically vindicated in light of the quantum Zeno paradox.

*Sojourn Time*

Zeno's paradox has also been found to appear in sojourn time, a topic in quantum mechanics which deals with the length of time a quantum system deals in a particular state. An example of one such paper dealing with sojourn time is one written by W. Jaworski (1989). Here Jaworski examines a system which is observed to decay in a certain quantum state, the one in which it exists for the observation. However, he mathematically concludes that should one constantly observe this system, then mathematically it would be found never to decay, thus supporting Zeno's proposal of a static universe.

*Supertasks*

The topic of supertasks also appears to exhibit paradoxical tendencies in place of Zeno's arguments. A supertask is a task which "requires an infinite number of operations to be completed in a finite amount of time" (Bokulich, 2003). Zeno is the first to be credited with considering the concept of a supertask with his Dichotomy argument, at least in theory. The paradox holds true in Zeno's classical argument, which assumes that it is impossible to complete the infinite series of tasks, and thus motion is impossible, as well as for the quantum variation of the argument, which assumes that completing the infinite series is possible, and thus motion is impossible. (Bokulich, 2003)

*Topology*

It is a bit shortsighted to pass off the work of Benardete and Prosser mentioned in Appendix III as mere puzzles. These puzzles are products of topology, the study of physical and mathematical surfaces. Topology is one of the most rapidly-increasing areas of mathematics in terms of interest and study. The most practical applications of this study involve technology. Though the technology possible is at present mere science fiction, Prosser (2006) highlights how some of the principles of infinity in surface study can lead to frictionless surfaces. These surfaces could be utilized for roads and safety features in automotives, and even items as mundane as frying pans, as the article's title hints.

*Motion and Time*

To even consider such business is subject to opening a Pandora's Box of conjecture and speculation. An entire library of books on the physics of time travel is in print for the reader who wants to understand why time travel is possible, theoretically.

As the relativistic view of physics dictates, time is a conception codependent upon motion. This principle is central to all modern debates regarding time, including: dilation, measurement, and even travel. It has become almost universally accepted that as motion exists, likewise does time progress. It follows logically that should motion cease, or be nonexistent, than either time stops or doesn't exist – else the relativistic understanding of the universe be wrong.

Yet, the relationship of motion and time was not introduced by Einstein as a by-product of relativity. The interdependence of time and motion may date back to as far as Aristotle, who wrote that "time does not exist without change" (218b21), and who concluded that time was something that, though not a type of motion as was a common view of the time, dependant upon motion. This view was cast aside by classical physicists, who sought to take off the Aristotelian robes that had prevented new discovery in the natural philosophies, the sciences, for over a thousand years. They viewed time and motion as absolutes that were independent of one another, and thought that the systems they had developed were sufficient to measure one quantity or the other with perfect accuracy, and better yet without the involvement of the other variable.

This illusion was shattered, however, after the discovery of relativity and the theories that have followed. The question of whether energy moves in quanta or continuously, or whether all outcomes are chance, formulaic outcomes, or by divine hands, is up for grabs. Yet the only consensus among these theories is that time and motion are necessarily dependent on one another: as time progresses, so must motion occur. If all motion ceases, then time stops, at least hypothetically.

The relationship of motion and time is pondered rather exhaustively by Adolf Grunbaum in *Modern Science and Zeno's Paradoxes* (1967). This book is the most comprehensive synthesis of scientific and mathematical theory with the philosophy of Zeno's paradoxes to date. Unfortunately, Grunbaum's book is now forty years removed from the forefront of scientific advancement, and was published long before the first research on Quantum Zeno effect, or any of the radical theories that are the progeny of advancements (that is to say, hypotheses) in quantum physics. A modernized tribute to Grunbaum's work is in due order, and will hopefully manifest itself sooner than later.

Among the most notable of these "radical theorists" is Julian Barbour, a highly respected physicist who in 1999 published *The End of Time: the Next Revolution in Physics*. In this highly provocative and widely acclaimed book he posits that the universe is timeless and static – certainly akin to doctrine of illusionary motion, which has been established to be a fundamental principle of Parmenides' school. Barbour explains how all sentient beings merely "travel" through a plane of forms in the most Platonic sense, a plane which he in fact calls "Platonia." By traveling, Barbour means merely to advance from one "frame" to the next, in a manner best illustrated by the still pictures of a film reel. The physics of how this reel moves composes approximately three-quarters of his book.

Barbour also mentions Zeno's arguments, particularly the Dichotomy (though Barbour mistakenly refers to it as the Arrow). Barbour explains that the arrow does in fact never hit the target, not because it would take an infinite amount of time (by Zeno's logic), but because the arrow shot from the bow and the one which hits the target are not the same arrow. (Barbour, 38-49)

## PSYCHOLOGY

In a practical society, one which seeks to unravel the mysteries of the universe, surely man must also search to understand the universe on a more personal level – the understanding of his own consciousness and existence and the biological mechanisms which drive them. Here is where psychology, the fledgling science, has arisen, so that humankind may proceed via the scientific method in the understanding of itself in the same way it searches to understand the cosmos. How does an archaic Greek philosopher fit in the equation of understanding a man's own mind? In reality, perceptual psychology's and physics' discoveries about the mechanisms through which people perceive the world are very akin to the teachings of Zeno's Eleatic school.

If the argument is to be made that the common sense idea of continuous motion cannot be proven by scientific evidence, and in fact science has found evidence that doesn't support continuous motion, rigorous examination of the human nervous system, especially the brain and eyes, must be undertaken. The connections that will be referred to may be found within the very workings of the human brain through the support of neurological evidence. The commission for making this argument is simple: each mechanism of the pertinent psychological apparatuses must be examined for characteristics that open the door for one to claim that the universe is not perceived biologically as it is colloquially.

*The Eye & Optic Nerve*

When asking about the mechanisms which drive human perception of motion, one must first look into the eye – where the visual information of the universe enters the observer. How fast can the eye perceive information? This is an area in which research is almost nonexistent. Why in such a heavily studied world is there a lack of research in such a seminally important area? Perhaps to understand the reason, first a summary of how the eye functions is in order. Light enters the eye through the retina, where light-detecting cells known as rods (for night-vision) and cones (for color vision) are stimulated. This stimulation is caused by a light-sensitive chemical in the rods and cones known as rhodopsin. Light causes the rhodopsin to change its molecular shape, which in turn causes the electrical charge in the rods and cones to change slightly- from -70mv to -40mv. This electrical charge then allows the information processed by the retina to travel along the optic nerve to the Lateral Geniculate Nucleus (LGN) of the brain, and then to the primary visual cortex where the information goes to numerous other areas of the brain. Because rhodopsin is readily available and the rods and cones are constantly producing an electrical signal, it is assumed that either the eyes perfectly detect (and to say *perfect* in this case is to refer to healthy eyes perceiving visual information at the rate and clarity with which it truly exists in the universe) visual information, or at least detect visual information at an incredibly high rate that does not merit concern for its limitation.

Though visual perception researchers do not apparently find this information particularly important, it is seminal in the argument which is being made. A well-supported claim may be made that human visual detection is not as "perfect" as many would assume it to be. The human eye contains a finite number of sense receptors: approximately 8 million cones and 120 million rods. In Evan Harris Walker's *The Physics of Consciousness* (2000), while discussing his theory on the bit rate at which the brain may present conscious data, which he named the "conscious channel capacity," some rather revealing information regarding the optic nerves' capabilities becomes apparent. The two optic nerves, which send information from the eyes to the brain, contain approximately one million nerve fibers each. These nerves transmit information to the brain by groups of pulses. The rate at which these pulses may travel peaks at approximately 1000 pulses per second: when a nerve fiber becomes completely saturated. Walker argues that this is not a practical value, however, because the optic nerves would be unable to sustain a level of activity that high for an extended period. He approximates the value of the pulse rate to be 200 pulses per second on average. These 200 pulses consist of 15-20 ten-pulse groups. Each of these ten pulses contains about 3.3 bits of information. Total, approximately 100 to 130 million bits of information may be sent from the eyes to the brain each second. In the previous section Walker discusses the "conscious field capacity," the amount of information which one observes in an instantaneous moment, i.e. the amount of information one could possibly perceive in a static image in a single instant. Walker hypothesizes the three-dimensional visual field to be approximately two million bits of information. Should one divide the capacity of the optic nerves to send information per second, 100-130 million bits, and the amount of information which may be perceived in a single instant, two million bits, one obtains a divisor between 50 and 65. What does this mean? With the ability of the brain aside, our eyes and optic nerves may only send between 50 and 65 static images to the brain in one second before saturating the optic nerves. Since humans are incapable of detecting all the details of the events which they believe happen in the universe, how then can a person be so sure that he or she is sensing continuous motion at all? This point will become much more compelling after discussion of the brain and the phenomenon of apparent motion.

*The Hopping Quantum Universe*

Consider the actual fluidity of the universe. Scientists currently use quantum physics to describe the universe on an atomic level. As discussed earlier, scientists find that the state of a system, i.e. the universe, is merely a series of probabilities until it is actually observed. Furthermore, they find that atoms move in packets of energy each known as a quantum (hence the name "quantum physics"). This pattern of movement is clear when one views atomic movements under a tunneling electron microscope. A person does not see electrons which move fluidly through space, but rather disappear in one instant and reappear in the next, giving the impression of "jumping" from one point in space to the next. If these elemental entities, from which the entire universe is comprised, move in a fundamentally discontinuous manner, how then can one argue for the perfect continuity of motion, when no continuous motion is seen on even the most elementary level?

*Looking Into the Past*

Should the benefit of the doubt be given to the general assumption regarding the performance of visual apparatuses, and it is assumed that human eyes have no inhibitions so far as to the time between which a stimulus is presented and the eye actually senses it, there must still be a finite limit on how close to the present the eye may receive information from the environment. This is because of two determining pieces of evidence: the measurable length of time it takes visual information to transit across the brain and the knowledge that light travels at a finite speed. Dealing with the former, the average neural pulse takes approximately 1/8 of a second to travel from the sense receptor to the critical area of the brain (Walker, 2000). So when someone sees a car whiz by him, he may not actually perceive it as directly in front of him until it has already passed by him/her. So far as the speed of light is concerned, it is an amazingly fast speed – 300,000 kilometers per second – yet it is a finite speed nonetheless. What this means is that should motion occur continuously, reflections at the speed of light are presented from the body undergoing the motion to the eye. Given this finite light-speed, one must conclude that it takes an amount of time for light to bounce from the body undergoing what is perceived as motion to the eye. What people merely view then are events which occurred a short time ago; humans view into the past. How far into the past humans are viewing is determined by the distance between the eye and the body being viewed (moving or not) divided by the speed of light, plus approximately 1/8 of a second.

This reality becomes more apparent when one gazes into the night sky. Many have heard the cliché that looking at the stars is "like looking into the past." This statement is true because of light speed's finite nature. When one sees a star that is 100 light years away, he/she is actually viewing that star when it was 100 years younger because of the length of time which it has taken the light to travel to Earth. Stars that are millions of light years away may have already burnt out and died, but man will continue to see them in the night sky until generation after generation has come and gone. There are those who argue that only the present exists and that the past is only a memory and the future is an illusion. But with this information, just the opposite is true. Not only does the past exist, but it is all that humans actually experience, and while seeing the past, the future is already occurring and is waiting to be perceived. Thus, only the past and future exist, and they exist simultaneously in relativity to the sentient observer.

*Man's Sluggish Brain*

Regardless of the eyes' potential capabilities and the nature of the speed of light, there is specific evidence that the human brain does not consciously perceive all information that the eyes send to it. This has been tested by numerous visual researchers using experiments in which a spot of light is flashed very quickly at a subject's eyes, and then the length of time in which the light is presented is adjusted, so that the absolute threshold at which the mind can consciously perceive the light stimulus can be determined. The general consensus which has been reached is that a stimulus must be presented for approximately 100 milliseconds before one may consciously perceive it. Despite the impressive powers of the eye, it is quite possible that our brains cannot detect on a conscious level perfectly fluid motion, but rather only a series of still images at approximately 100 milliseconds between each one.

*Gestalt Theory & Apparent Motion*

How then may people perceive a seemingly continuous motion if that is not what the human brain actually interprets from the eyes? Humans are able to perceive this fluid motion because of a psychological phenomenon known as apparent motion. This phenomenon was first studied in 1912 by Max Wertheimer. He discovered that a series of still pictures, when shown to a subject in rapid succession, could give the illusion of continuous motion. At what threshold did he find that humans begin to perceive continuous motion from still-frames shown in rapid succession? The answer is around 15-20 frames per second (Walker, 2000). From these findings Wertheimer became the father of the psychological revolution known as Gestalt Theory: the basis of this theory being that the whole is greater than the sum of its parts; that is, people may perceive a sum of static images as not a mere slideshow but rather a moving scene. This is also the same concept upon which film and television function. Film shows project a series of static pictures at 24 frames per second (just above the conscious threshold of apparent continuous motion given above) at the eyes, and the United States' NTSC television signals refresh on American television screens at a rate of 30 frames per second. Those who view these broadcasts perceive these pictures as the actors, anchormen/anchorwomen, and sports heroes of the big and little screen.

Another question then arises: if motion is merely a contrivance of the human brain, then may humans suffer brain damage which inhibits only the experience of motion? The answer is yes. There is a documented disorder which impairs the experience of motion – clinically defined as motion agnosia. This disorder results from damage to the medial temporal (MT) area of the brain: the area which was aforementioned to be responsible for our perception of motion. The effect of this disorder is straightforward – a loss of movement perception in all of the three directions (or the three perceived dimensions of space). Patients suffering from this disorder have been documented as having symptoms such as inabilities to pour coffee because it appears frozen and follow dialogue because facial features can not be seen. Furthermore, when with other people in a room, the other people seem to "jump" from place to place, being at one location in one instant and somewhere else in the next. (Zihl et al., 1983, 1991)

*Summary*

The point to be made here is a philosophical one. Ask the layperson why motion exists, and he/she will not recall the results of some famous physics experiment, or begin down a line of complex reasoning. Rather, he/she will simply say “I believe it exists because I can move right now – I experience it.” This is far and away the most common argument; after all, if science has taught civilization nothing else, it has instructed individuals to make the most reasonable conclusion based on observational evidence. The connection is simple: I experience motion both in myself and from the environment; therefore, motion must exist. The previous evidence then is a direct attack upon this notion for several reasons. Firstly, in light of quantum physics humans do not observe the universe as moving continuously on the atomic level. Secondly, the human eye is not only limited in the detail it is capable of sensing, but is also limited in the ability by which it may see the present in the universe, if not by possible inhibitions in the eye’s biological mechanisms, then at least by the finite nature of the speed of light. From there the optic nerve is characteristically inhibited by its limited capacity to send information, as well as the delay it suffers in changing its electrical potential with each pulse. Humans also have minds that do not perceive all the information which is presented to the eye and sensed, whatever amount that is, as well as minds which are capable of taking still images and perceiving them as continuous motion when the images are presented quickly enough. The conclusion which must be reached from this information may be posed in the form of a question: how can a person honestly conclude the existence of a continuously moving universe, when not only have scientists not observed one on a basic physical level, but given the imperfections of the eye and brain, no person can never truly say that he/she's experienced one either? Zeno's school of thought taught that motion was merely an illusion of human perception; it is strongly evident that continuous motion may indeed be nothing more. The motion which man concludes so confidently is quite possibly merely a belief. Not through science but through faith does motion occur continuously – the same faith which is exercised when men take for granted having the breath of life the following day, the same faith men exhibit when they proclaim the existence of their gods.

## THE GREAT CONTROVERSY

The value and validity of Zeno's paradox have been argued over, despite these applications to his problem. It is simple to kick Zeno to the curb and say that his problems have been solved so physicists may work on harmonizing relativity and quantum mechanics (or something of the sort). Individuals such as Andrew Krywaniuk feel that not only is Zeno's paradox not valid, but he is not a particular fan of Zeno's philosophical tradition as is evident in his article "The Failure of Philosophy" (1997). It does not take much imagination for one to assume that there are many who share such a similar view, and such an assumption is correct. It is very easy, sometimes to a fault, to accept a pragmatic philosophy. After all, what practical good does dabbling in strange metaphysical concepts do for humankind? It is clear humans experience motion in their everyday lives. Why then should one be concerned whether or not it scrupulously follows all scientific findings? Such an argument is not only trying to take an easy way out, but is completely anathema to the primary objective of science – to discover truths about physical reality. With this said, there are also many who still argue that not only is Zenonian logic and physics applicable today, but those who even say that Zeno's paradox, in its original text, is correct. One example of a published paper dealing with this is "The Tortoise Is Faster," by Constantin Antonopoulos (2003). A firm supporter of Zeno's genius and the value of his arguments is Bertrand Russell, one of the most prolific philosophers and mathematicians of the twentieth century. In his book *Our Knowledge of the External World*, Russell goes as far as to say: "Zeno's arguments, in some form, have afforded grounds for almost all the theories of space and time and infinity which have been constructed from his day to our own" (Russell, 1926, p. 183). There are certainly others who share comparable views. It is difficult to find much common ground among those who continue to study, argue, support, and refute Zeno's ancient arguments.

## Purpose

With all of the argumentation surrounding Zeno's paradoxes, perhaps it would be clarifying to look to the source itself for enlightenment as to why Zeno constructed these arguments: Zeno's original writings. Unfortunately, a rather hindering road block stands in the way of accomplishing such a task – Zeno's writings have never been discovered. The only record of Zeno available is primarily through an indirect account of Aristotle, and supporting considerations from Plato, Simplicus, and Proclus. However, one may deduce a handful of rather feasible theories regarding Zeno's motives.

The first is that Zeno genuinely wanted to disprove motion. As it has been established, a fundamental idea of the Eleatic school of thought was that motion was merely a human perception, and that the universe is static. It is also known that Parmenides was a founding father of what Aristotle eventually formalized as logic. Given these two facts, it would seem logical that Zeno would put these pieces together so to speak in order to create logical arguments to support his teacher's beliefs, which were highly controversial even in ancient Greece.

The second theory is that Zeno merely meant these arguments in jest. The arguments were published without prior consent from Zeno[7], and it may not have been

[7] At least one must assume so, since there is no known surviving copy of any of Zeno's work. The first known thinker to give consideration to Zeno's Paradoxes, Aristotle, lived in a time period too far removed from the time of Zeno of Elea to have even known him, much less ask permission to exhibit his arguments in his *Physics*.

Zeno's intention for these arguments to ever see the light of day. It was fashionable during this time to use logical argumentation, which Parmenides used, to refute the very ideas Parmenides taught. Thus, Zeno, wanting once again to defend his teacher, created his arguments to serve as a satire of those arguments being placed before Parmenides. It would certainly fly in the face of Eleatic detractors if Zeno could say, "Look, I've used the same method you use to attack us to show that motion isn't even possible! What say you now?" Regardless of what Zeno's motives were though, his arguments did survive, and they have been a thorn in the side of, or a stimulus to thought for, those who seek to ponder logically for 2,500 years.

A third possibility is that Zeno's original intention for the paradoxes was the purpose that they have served for science: as tools by which to create more consistent models of reality. It is often by revealing what is wrong with a theory that it may be made more reasonable, more perfect. As Eric Engle points out, "paradoxes exist to point out flaws in our reasoning" (p.1). Zeno may have been aware that the common-sense notion of physics during his time was not accurate, and by constructing his paradoxes he could challenge future generations of thinkers with more accumulated knowledge of nature to answer his arguments, and in turn make human understanding of physical reality more comprehensive. Grunbaum makes this speculation about Zeno's paradoxes of extension when he states that "Zeno challenged geometry and chronometry to devise rules for adding lengths and durations which would allow an extended interval to consist of unextended elements" (1967, p. 3). Though this calls for a certain degree of faith in Zeno's farsightedness, the testament to Zeno's intellectual acumen has been made evident by not only this author, but almost the entire host of Western philosophical greats.

## Conclusion

Much has been argued about Zeno and his little arguments. They have seemingly been refuted and proven by numerous individuals on mathematical and philosophical grounds ever since circa 500 B.C. Even with our modern relativity, Minkowskian space, and infinite-series convergence, academia as a whole still cannot come to a final consensus over the validity of this paradox. However, perhaps closure is not necessary. As a professor commented to me, “most interesting philosophical questions don’t converge on unanimity. They’re not like inquiry into the boiling point of fluids.” We find that, upon inspection, few questions in philosophy ever reach the convergence that would be satisfying to us as thinkers, or even as human beings.

Regardless of the validity, however, the fruits have been apparent. Many innovative ideas in physics and philosophy (as well as psychology, mathematics and numerous other subjects) have come from the pondering of Zeno’s arguments. These arguments have also led to a great refining of logical argumentation and scientific rigor throughout the millennia. For these reasons alone, we should conclude that Zeno’s paradox has been highly important to both physics and philosophy, being a theme common among many topics in both fields. Whether the paradox is right or wrong hasn’t really seemed to matter.

The question with which the reader, as well as the author, is faced is a question of validity.  Though Jonathan Barnes tells us that "Zeno now stands as the most celebrated of Pre-Socratic thinkers," (1996, p. 231) to what degree are these arguments beyond the novelty of studying archaic philosophy?  Is this consideration of Zeno's paradoxes one deserving of further study, or are they and the theories used in support of this paper mere foolishness?  Why do those who either now do or continue steadfast in their belief of this paradox's validity believe?  We must question whether this is a belief founded in reason and the furthest progression of good science, or but a wish for the existence of "scientific mysticism," so to speak  As Ronald de Sousa writes of Zeno's Paradoxes: "Their lasting appeal comes from the fact that they appear to give us reason to question something" (de Sousa).  What the reader may now be struggling with, as this author has continued to struggle with since the beginning, is whether or not this paradox lives on as a means of psychological satisfaction in that we are given an opportunity to question something fundamental about the world.  Can it be so simple an answer?  Whether we are delving into science or astrology, however, is an answer which must ultimately be made by the reader.  Perhaps the reader is disgruntled by such a conclusion; shall I end on clichéd statements of postmodernism: that the only reality and truth is that in the mind of the individual?  Is the primary constituent of truth the same which Grunbaum and others claim is of becoming, and that which a myriad of people since the first Ionian philosophers have argued is the basis of ethics: the evolved mind?   Unfortunately, I find that I am compelled to.  The reader may find some consolation from science, however. Let us consult quantum physics; if the theoretics of quantum physics are accurate (as this author and many others certainly hope), then such a reality is indeed the only reality which exists - the reality which each conscious being creates in his/her mind.  In this spirit, as long as we may justify our beliefs with reason let us believe what we may: for the building blocks of the universe are not matter and energy but our own disembodied psyches.

Finally, let us not forget what is perhaps most brilliant of all about Zeno and his paradox. A philosopher who lived 2,500 years ago showed that the classical concepts of space and time and the way in which they fit together do not logically work, and by virtue of this he in some fashion anticipated principal theories of science and mathematics far ahead of his time. It has taken a plethora of geniuses two and a half millennia to produce the scientific and mathematical theories to sufficiently answer Zeno's questions - these theories being the most compelling and profound of their respective ages and disciplines. A man such as this is someone to whom we should all tip our hats.

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## Appendix I

Proof for the irrationality of the square root of 2

1. For the square root of 2, defined as R, to be a rational number, it must be the divisor of 2 numbers in lowest terms, defined as p and q. If this assumption leads to a contradiction, then it is necessarily true that this assumption is false (that the square root of 2 is rational).

2. The above statement may be written as $R = p/q$.

3. The equation in (2) may be squared, so that $R^2 = p^2/q^2$.

4. Since R is the square root of 2, $R^2 = 2$, which may replace $R^2$ in the equation.

5. Hence $2 = p^2/q^2$, or $p^2 = 2q^2$.

6. It follows then that p must be an even number, because it has a factor of 2.

7. So $2x = p$, where x is some integer with value $n^2$.

8. So if p is replaced by 2x in the equation in (5), $2q^2 = (2x)^2$, or $2q^2 = 4x^2$.

9. Divide both sides by 2, and one gets $2x^2 = q^2$. So q is even since it has a factor of 2.

10. If q and p are both even (because they both have a factor of 2), the fraction is not in lowest terms.

11. Since this contradicts the original assumption, the assumption that such a rational number exists is wrong.

**Appendix II**

Examples of Limit Theory (Kouba, 1996)

**I.**

$$\lim_{x \to 3} \frac{5x^2 - 8x - 13}{x^2 - 5} = \frac{5(3)^2 - 8(3) - 13}{(3)^2 - 5} = \frac{8}{4} = 2$$

**II.**

$$\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4} = \text{''}\frac{0}{0}\text{''} = \lim_{x \to 2} \frac{(x-2)(3x+5)}{(x-2)(x+2)} = \lim_{x \to 2} \frac{(3x+5)}{(x+2)} = \frac{3(2)+5}{(2)+2}$$

$$= \frac{11}{4}$$

**III.**

$$\lim_{x \to 1} \frac{x^{\frac{1}{3}} - 1}{x^{\frac{1}{4}} - 1} = \text{''}\frac{0}{0}\text{''} = \lim_{x \to 1} \frac{(x^{\frac{1}{3}} - 1)}{(x^{\frac{1}{4}} - 1)} \frac{\{(x^{\frac{1}{3}})^2 + (x^{\frac{1}{3}})1 + (1^2)\}}{\{(x^{\frac{1}{3}})^2 + (x^{\frac{1}{3}})1 + (1^2)\}}$$

$$= \lim_{x \to 1} \frac{x - 1}{(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)(x^{\frac{1}{4}} - 1)}$$

$$= \lim_{x \to 1} \frac{x - 1}{(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)(x^{\frac{1}{4}} - 1)} \frac{(x^{\frac{1}{4}} + 1)((x^{\frac{1}{4}})^2 + (1)^2)}{(x^{\frac{1}{4}} + 1)((x^{\frac{1}{4}})^2 + (1)^2)}$$

$$= \lim_{x \to 1} \frac{(x-1)(x^{\frac{1}{4}} + 1)(x^{\frac{1}{2}} + 1)}{(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)(x - 1)} = \lim_{x \to 1} \frac{(x^{\frac{1}{4}} + 1)(x^{\frac{1}{2}} + 1)}{(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)} = \frac{((1)^{\frac{1}{4}} + 1)((1)^{\frac{1}{2}} + 1)}{((1)^{\frac{2}{3}} + (1)^{\frac{1}{3}} + 1)}$$

$$= \frac{4}{3}$$

**VI.**

$$\lim_{x\to 0} \frac{x^3 - 7x}{x^3} = {}''\frac{0}{0}'' = \lim_{x\to 0} \frac{x(x^2 - 7)}{x(x^2)} = \lim_{x\to 0} \frac{x^2 - 7}{x^2} = {}''\frac{-7}{0^+}'' = -\infty$$

## Appendix III

Modern Puzzles Based upon Zeno's Arguments

Many minds since Zeno have created paradoxes of their own for thinkers to mull over. Some of the most famous paradoxes in modern times have been the puzzles constructed by Jose Benardete, in his book *Infinity: An Essay in Metaphysics* (1964). The most notable of these puzzles are noted below.

- "The Paradox of the Gods" (Laraudogoitia, 2000). The basic concept of the argument is that a series of "gods" place walls in front of a man's way, doing so in decreasing intervals along his path (1/2 of the way, 1/4 of the way, 1/8 of the way, etc.). If this continues to infinity, even though there is never a wall put up by the "gods" the man is still never able to move; the man can never move because of the wall that will be placed in front of him, but because he never moves a wall is never placed in front of him. Priest (1999), Laraudogoitia (2000), Yablo (2000), and others have all made their own observations on this paradox, arriving at different conclusions and offering alternatives and augmentations.
- "The Book Paradox." Benardete describes a book, with each page being half the width of the previous page. So, the first page has a thickness of one unit, the second page ½ unit, the third ¼ unit, and so on. Assuming that there is no bottom limit on the possible thickness of the pages, the book may have an infinite number of pages, though its width will be no greater than two units (by virtue of the

> convergence of the infinite series in calculus). The study of such theoretical objects is involved when discussing the mathematical principles of topologically open and closed surfaces, that is, surfaces that either have "no outermost layer of points" (Prosser, 2006, p.188) or have an outermost layer of points. Simon Prosser (2006) expands on the theoretical questions surrounding such a book in his article *The Eleatic Non-Stick Frying Pan*. He asks a series of questions about the topologically open surface of the book such as: could a piece of tape could stick to the surface, what would happen to a ball should it be dropped on the surface, and could a suction cup be applied to the surface to lift the book. He answers all of these questions with deductive reasoning beginning with the physical properties of the forces involved in such actions, and with the accompanying mathematics.

Mathpages (2006) has taken Zeno's Dichotomy and reworded his analogy to make the argument applicable again. Though the theories discussed previously, particularly limit theory, provide compelling evidence that Zeno's infinite series converges into a finite distance, the argument may be slightly altered to address another paradox, one which involves the convergence of a geometric rather than infinite series. They ask, what if a photon bounces off a series of mirrors in an ever-decreasing spiral, a "Zeno's maze?" Though the spiral does have a finite size, bounded by the outer bounds of the spiral (and because of this the photon must leave the spiral), one cannot tell in which direction the photon will travel out of the spiral because there is no final mirror for the photon to bounce off, and the photon never escapes. A convergence of an infinite series into a finite number is not a satisfactory explanation for this puzzle, as the maze is already assumed to be finite in size, given that it has a known perimeter.  The only seemingly logical answer to this resulting paradox, given the present knowledge of physics and mathematics, is that the universe must be composed of a smallest indivisible particle so that there is a last mirror, one which is composed of the alleged single indivisible particle.