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Magnetohydrodynamics in an Open Universe

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1980

MAGNETOHYDRODYNAMICS IN AN OPEN UNIVERSE

A Thesis

Presented to

the Faculty of the Department of Physics and Astronomy

Western Kentucky University

Bowling Green, Kentucky

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

by

Oscar L. Norris

December, 1980

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MAGNETOHYDRODYNAMICS IN AN OPEN UNIVERSE

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Acknowledgements

The writer expresses his thanks to his Mother and to his Father's memory without whose deprivation his educational goals could not have been realized. The writer also thanks his wife, Nell, his brothers, Art and Jim, and their families for their help and support. Thanks must also go to one other person, for which words cannot express his full gratitude: without the loyal belief, support, and encouragement of my advisor, this thesis would not have been completed.

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Magnetohydrodynamics in an Open Universe

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A study of magnetohydrodynamics (MHD) in an open universe is presented. We discuss the data and justification behind our choice of an open universe for investigation in this thesis, as opposed to the closed universe theory, which is more appealing in some ways. We explicitly define all parameters used in the analysis of magnetohydrodynamic Bianchi Type V cosmologies and outline the formulation behind them. We then proceed to present a solution to a Bianchi Type V magnetohydrodynamic cosmology with a diagonal metric. After that, the results are compared with present observations. Lastly, we conclude with an assessment of the model and discuss areas for future work, such as nondiagonal metrics and the role of perturbations of the models in galaxy formation.

CHAPTER I. INTRODUCTION

Recent Cosmology

For several years there has been much discussion, arguments, and even strong disagreements over the question of the origin of the universe.[1] Has it always been as it is? Will it be as it is for an eternity?[2] Did it start with a big bang[3] or a big whimper?[4] Is it expanding?[5] If it is, will it continue to expand or will it, at some point, start collapsing?[6] Most of these questions have surfaced in recent history as our technology and therefore experimental understanding of natural phenomena has risen exponentially.[7] But even with such exponential growth of our understanding, we still don't know all the answers. A large portion of present theory and observation support the big bang theory.[3, 8-10] Since we have a generally accepted idea of how it started, we next ask: is the universe open or closed?; i.e., will it expand forever or recollapse? In spite of the appeal of the closed universe,[11,12] the evidence seems to be in favor of an open universe[13] as we will discuss shortly.

Other questions concern the isotopy of the universe.[14] (uniformity of observations in all directions) and the homogeneity of the universe (independence of observation on position).[5,15,16]

Friedman Models

The homogeneous Friedman Model of the universe is a very widely accepted model along with others that have slight modifications, (i.e., coordinate transformations) such as the homogeneous and isotropic Friedman-Robertson-Walker model.

The equation for the shortest line between two points or the metric of this spatially isotropic and homogeneous model would be of the form:

$$ds^2 = -dt^2 + R^2(t)\left[dx^2 + \Sigma^2(x)(d\Theta^2 + \sin^2\Theta d\phi^2)\right] \tag{1}$$

This can, and will be, expressed in a new notation[17,18]

$$ds^2 = -dt^2 + e^{2\alpha} e^{2\beta}{}_{ij}\sigma^i\sigma^j \tag{2}$$

where the σ^i are a basis of differential forms:

$$d\sigma^i = -\tfrac{1}{2}c^i{}_{jk}\sigma^i \Lambda \sigma^k \tag{3}$$

For the Friedman model B_{ij} = o and $e^{2\alpha}$ $R^2(t)$ in equation (1)

n a Bianchi type I or flat 3-space model, the metric is:

$$ds^2 = -dt^2 + R^2(t)\left[dx^2 + \Sigma(x)^2(d\Theta^2 + \sin^2\Theta d\phi^2)\right] \tag{4a}$$

or

$$ds^2 = -dt^2 + e^{2\alpha}\delta_{ij}dx^i dx^j \tag{4b}$$

A Bianchi type IX closed space metric is:

$$ds^2 = -dt^2 + R^2 dx^2 + R^2\sin^2 x(d\Theta^2 + \sin^2\Theta d\phi^2) \tag{5a}$$

or

$$ds^2 = -dt^2 + e^{2\alpha}\sigma^i\sigma^j \tag{5b}$$

and lastly for a Bianchi type V space, the metric is:

$$ds^2 = -dt^2 + R^2 dx^2 + R^2\sinh^2 x(d\Theta^2 + \sin^2\Theta d\phi^2) \tag{6a}$$

or

$$ds^2 = -dt^2 + e^{2\alpha}(dx^2 + e^{2x}dy^2 + e^{-2x}dZ^2) \tag{6b}$$

in our new notation and cartesian coordinates.

In type I $c^i{}_{jk} = 0$ implies a spatial curvature K = 0 operator, for type IX $c^i{}_{jk} = \epsilon^i{}_{jk}$, the three dimensional permutation operator implies K = 1, and for a type V $c^i{}_{jk} = 1$ (where i = j = 2, 3 and k = 1) implies K = -1.

For all calculations in this brief review, the cosmological constant is set to zero. For FRW models, Einstein's field equations become

$$G^{\mu\upsilon} = 8\pi G T^{\mu\upsilon} \tag{7}$$

and the stress-energy tensor is for a perfect fluid

$$T^{\mu\upsilon} = (\rho + P)\, u^{\mu}u^{\upsilon} + Pg^{\mu\upsilon} \tag{8}$$

where ρ is the energy density and P is the isoptropic pressure and u^{μ} is the 4-velocity. The contracted Bianchi identifies

$$G^{\mu\upsilon}{}_{||\upsilon} = 0 \tag{9}$$

where '$||$' is the corvariant. This implies that

$$T^{\mu\upsilon}{}_{||\upsilon} = 0 \tag{10}$$

which is just the equation of conservation of inertia. The equation of state is

$$P = P(\rho) \tag{11}$$

The Einstein equations for the metrics (4) - (6) are

$$3\dot{\alpha}^2 + \tfrac{1}{2}R^* = T^{00} \tag{12a}$$

$$R_{oi} = 0 = T_{oi} \tag{12b}$$

$$-6\ddot{\alpha} - 9\dot{\alpha}^2 - \tfrac{1}{2}R^* = T^{k}{}_{k} \tag{12c}$$

$$0 = T_{ij} - \frac{1}{3}\,\delta_{ij}T_{kk} \tag{12d}$$

where $R^* = -6e^{-2\alpha}$ for $K = -1$, 0 for $K = 0$ and $3e^{-2\alpha}/2$ for $K = +1$. In the equations, (12a) relates expansion to curvature and inertia density, (12b) confirms that there is no momentum flux (fluid circulation), (12c) is the evolution equation for $\dot{\alpha}$, and (12d) confirms that there are no tracefree stresses in the Friedman models. For these models, then

$$U_u = \delta^0{}_u \tag{13}$$

and so

$$T^{oo} = \rho,\ T^{k}{}_{k} = 3P \tag{14}$$

For dust P = 0, and for radiation $P = \frac{1}{3}\rho$. The conservation equations are then

$$\frac{d}{dt}(\rho_d e^{3\alpha}) + \frac{d}{dt}(P_r e^{3\alpha}) + \frac{1}{3}\rho_r \frac{d}{dt}(e^{3\alpha}) = 0 \tag{15}$$

If the fluids do not interact (a reasonable assumption over most of the later history of the universe) then Equation (14) gives

$$\rho_d e^{3\alpha} = \rho_{d_o} e^{3\alpha_o},\ P_r e^{4\alpha} = \rho_{r_o} e^{4\alpha_o} \tag{16}$$

where subscript 0 indicates a constant. The parametric solutions can be written to Equation (11) using Equation (15) as

$$\left.\begin{aligned} e^{\alpha} &= R = \delta(1-\cos\xi) + \beta + \xi \\ t &= \delta(\xi - \sin\xi) + \beta(1-\cos\xi) \end{aligned}\right\} \quad K = +1 \tag{17a}$$

$$\left.\begin{aligned} e^{\alpha} &= R = \tfrac{1}{2}\delta\xi^2 + \beta\xi \\ t &= \frac{1}{6}\delta\xi^3 + \tfrac{1}{2}\beta\xi^2 \end{aligned}\right\} \quad K = 0 \tag{17b}$$

$$\left.\begin{aligned} e^{\alpha} &= R = \delta(\cosh\xi -) + \beta\sinh\xi \\ t &= \delta(\sinh\xi - \xi) + \beta\cosh\xi - 1 \end{aligned}\right\} \quad K = -1 \tag{17c}$$

where $\delta = \rho_{d_o} e^{3\alpha_o}$ and $\beta^2 = \rho_{r_o} e^{4\alpha_o}$.

For use in further discussion, we define the following frequently used quantities:

The Hubble parameter H:

$$H = \frac{\dot{R}}{R} \text{ or } \frac{\dot{e}^{\alpha}}{e^{\alpha}} = \dot{\alpha} \tag{18}$$

The deceleration parameter q:

$$q = -\frac{\ddot{e}^{\alpha}}{e^{\alpha}H^2} = -\frac{\ddot{R}}{RH^2} \tag{19}$$

The density parameter Ω:

$$\Omega = \frac{8\pi G\rho}{3\ H^2} \tag{20}$$

and from Einstein's equations Ω can be written as Ω = 2q and $K = H^2(e^{\alpha})^2\ (\Omega - 1)$.

In many observational discussions, one uses those parameters. For the special case of K = 0 (or flat 3-space) Ω = 1 and $q = \frac{1}{2}$;

these are critical values between eventual collapse $K = +1$ and perpetual expansion $K = -1$. ρ for $\Omega = 1$ is frequently written ρ_c for the "critical density" of $\Omega_c = \rho/\rho_c = 1$.

CHAPTER II. OPEN OR CLOSED UNIVERSE?

Mean Luminosity and Density Enhancement

One school of thought of present day cosmology maintains that the universe is closed.[12] There are several approaches that indicate the universe is not closed, but open. There are also many papers on these topics; therefore, there is a reliable conviction that their methods are valid. An analysis of the mean luminosity density of galaxies gives critical density Ω of less than one.[20]

In this method $\Omega \simeq \Omega_G$ where Ω_G is the contributor to Ω from matter associated with galaxies. Determination of Ω_G requires the determination of two parameters: the mean luminosity density in the universe and a characteristic mass to light M/L ratio. Then

$$\Omega_G = \frac{8\pi G}{3H_o{}^2}\, \rho_L \; M/L \tag{21}$$

The mean luminosity density ρ_L can be found via

$$\rho_L = \frac{6}{\sqrt{\pi}} \frac{NL^*}{Ad^3} \tag{22a}$$

where $\rho_L = L^*\phi^*$ and $N = \frac{1}{6}\pi^{\frac{1}{2}} A\phi^* d^3$

$$\phi\frac{(L\ )}{(L^*)}\ d\frac{(L\ )}{(L^*)} = \phi^*\frac{(L\ )}{(L^*)}\exp(-L/L^*)d\frac{(L\ )}{(L^*)} \tag{22b}$$

and ϕ^* is a normalization constant and $L^* = 3.4 \times 10^{10} L_\Theta$ and N = number of galaxies in a solid angle A, giving a value of[20]

$$\rho_L = 4.7 \times 10^7 L_\Theta \mathrm{Mpc}^{-3} \tag{22c}$$

There is much discussion about the contribution of halos to the characteristic mass to light ratio, which has been studied by Gott and Turner and determined not to be of extreme importance.[20] An estimate of $M/L = 120\ M_\Theta/L_\Theta$ seems to be viable.[20]

An analysis of perturbations in the Hubble flow induced by density enhancements of the distribution of matter supports a value of Ω less than one.[21] The authors reason as follows: Consider the evolution of a density perturbation, beginning with a density contrast $\delta\rho = \gamma\rho$ where γ is a function of time. If the condensation is bound, its Hubble expansion will turn into collapse and γ will grow. If it's unbound, its Hubble flow will only be retarded while γ approaches an asymptotic value. In redshift space (momentum space), clumps which have not significantly collapsed will be undistorted, while for a large slowing in the Hubble flow, there will be a large distortion. Regardless of distortions in redshift space, any density perturbation in configuration space will also show a density enhancement in redshift space. The enhancement γ is a function of Ω.

Consider two galaxies with spherical polar coordinates $(\theta_1, \phi_1, CZ_1/H)$ and $(\theta_2, \phi_2, CZ_2/H)$, with angular separations Δ_{12}. Then their redshift space separation d_{12} is

$$d_{12} = \frac{C}{H_o}\left[Z_2{}^2 + Z_1{}^2 - 2Z_1Z_2 \cos\Delta_{12}\right]^{\frac{1}{2}} \tag{23}$$

which has the projection ℓ_{12} on the celestial sphere of

$$\ell_{12} = \frac{C}{H_o}(Z_1 + Z_2) \tan\left(\frac{\Delta 12}{2}\right) \tag{24}$$

The angle α between the separation vector d_{12} and the target plane of the sky at the midpoint between them is $(Z_1 \geq Z_2)$

$$\alpha = \tan^{-1}\left[\tfrac{1}{2}(Z_1/Z_2 - 1) \cot\left(\frac{\Delta 12}{2}\right)\right] \tag{25}$$

If the Hubble flow is unperturbed, the mean value of α is $\alpha = 32^o.7$. If H is merely slowed $\alpha < 32^o.7$, and for a bound region $\alpha > 32^o.7$.

The density enhancement in any region of redshift space can be determined given an accurate and large enough sample of redshift data

$$\alpha^{i}(D) = \frac{1}{N_i(\leq D)} \sum_{j=1}^{N_i(\leq D)} \alpha_{ij} \tag{26}$$

and

$$\zeta_i(D) = 4\pi/A\frac{(CZmax)}{(H_o)}^3 N^{-1} N_i D^{-3} \qquad (27)$$

where N is the number of galaxies brighter then a given minimum luminosity L A is a solid angle defining a volume in redshift space to a maximum Z. d_{ij} is redshift space separation, ℓ_{ij} is projected separation, α_{ij} is the separation vector angle, and $\zeta(D)$ is the mean density enhancement, and the d_{ij}'s are a set smaller than or equal to a maximum D for the i^{th} sample galaxy. Combining Equations (26) and (27) gives

$$\left\langle \langle\alpha\rangle \; (\zeta) \right\rangle = N^{-1} \sum_{i=1}^{N} \langle\alpha\rangle^{i} (\zeta) \qquad (28)$$

If we start from a uniform expansion then

$$\gamma \tan \alpha_I = \zeta \tan \langle\alpha\rangle \qquad (29)$$

where α_I is $\langle\alpha\rangle$ for an isotropic, undistorted, distribution of galaxies. This means that

$$\langle \tan \alpha \rangle = \frac{(G\rho)^{1/2}}{H} \qquad (30)$$

which can be related to Ω directly from Equation (20):

$$\Omega = \frac{8\pi}{3} \langle \tan \alpha \rangle^2 \qquad (31)$$

Sargent and Turner apply their argument (just outlined above in Equations (23) - (31) using the Uppoala General Catalogue[22] to find a present most likely value $\Omega \approx 0.07$.

Mean Density

A study of the mean density associated with galaxies by Seldner and Peebles lends strong support to a value Ω of less than one.[23] The authors claim as follows: a dimensionless function $\frac{n(r)}{n}$ is estimated, where n(r) is the mean number density of galaxies at a distance r from an Abell cluster of given richness class[24] and n is the galaxy number density averaged over the region of space surveyed by Shane-Wintamen.[25] If r is not too small, so $\frac{n(r)}{\langle n \rangle}$ is not greatly different from one, the number density ratio should approximate the corresponding ratio of mass densities

$$\frac{\rho(r)}{\langle\rho\rangle} = \frac{n(r)}{\langle n\rangle} \tag{32}$$

(i.e., the galaxy distribution at r approximates representative sample of the mass distribution.) At small enough r the clusters are thought to be in dynamic equilibrium.

One may estimate the $\frac{n(r)}{\langle n\rangle}$ function via the following process, consider the equation

$$N(\Theta) = \langle N\rangle\left[1 + W_{gc}(\Theta,D,R)\right] \tag{33}$$

where $N(\Theta)$ is the mean count of galaxies per unit solid angle at angular distance Θ from an Abell cluster center, averaged over clusters of a chosen distance R and richness class D. W_{gc} is the cross-correlation function from Shane and Wintamen.[25] Place at the known angular position of each Abell cluster a symetric galaxy distribution

$$N(\Theta) = \langle N\rangle A_{DR}\Theta^{-\tau},\ \Theta < \Theta_D \tag{34a}$$

$$N(\Theta) = 0,\ \Theta > \Theta_D \tag{34b}$$

where A_{DR} is an adjustable amplitude (function of D and R) and add a uniform background to make up the observed N . This produces a model $W_{gc}(\Theta,D,R)$ that varies as $\Theta^{-\tau}$ at small Θ and fluctuates and drops less rapidly at large Θ due to clusters seen nearby in projection, both accidentally and correlated. Variations of the amplitude A_{DR} with distance provides information about the galaxy luminosity function and the assumption that $\frac{N(r)}{\langle N\rangle}$ is independent of the absolute magnitude down to which one counts (to which one can see). The A_{DR} are well fitted by the Abell form for the number $N(<M)$ of galaxies brighter than a given magnitude M:

$$N(<M) \propto \mathrm{dex}\left[\alpha(M-M^*)\right] \quad , \quad M<M^* \tag{35a}$$

$$\propto \mathrm{dex}\left[\beta(M-M^*)\right] \quad , \quad M>M^* \tag{35b}$$

where $\alpha = 0.80, \beta = 0.10$, $M^* = -18.3 + 5 \log h$, and $h = \dfrac{H_o}{(100 \text{ Km sec}^{-1}\text{Mpc}^{-1})}$. Now the relationship between surface density (equation (33)) and space density around the cluster depends on the luminosity function. Using equation (35) Seldner and Peeble find that

$$\frac{N(r)}{\langle N \rangle} = (Bf_R) \;/\; (r^{2.4}) \tag{36}$$

where $B = 165h^{-2.4}$, f_R is the richness function,[24] and $0.5<hr<15$Mpc. Changes in the luminosity function that still permit a reasonable fit to A_{DR} can change B by at most 20%. The mass density in a manner analagous to the preceeding section can be estimated by

$$\rho(r) = \sigma^2/(2\pi G r^2) \tag{37}$$

where σ is the velocity despertion in the line of sight and G is the gravitational constant. (This is the virial theorem.) Combining equations (37) and (36) gives

$$\Omega = 4\langle\sigma^2\rangle r_j^{\,0.4}/3H^2 f_R B, \tag{38}$$

where $r_j = 2h^{-1}$Mpc. Using this equation and the data from 12 clusters, they find the mean density parameter Ω is

$$\Omega = 0.69 \tag{39}$$

<u>Peculiar Velocity Field</u>

Study of the peculiar velocity field in the local supercluster[26,27] implies that it is not unreasonable to expect Ω less than one.[28]

Peebles uses a spherical model, the virgo cluster. It is assumed that the number density of galaxies varies with proper distance d from the center of the virgo cluster as

$$N(d) = \langle N \rangle (1 + A/d^{\gamma}) \tag{40}$$

where $\gamma = 2$ and N is the large scale mean density of galaxies. The parameter A is adjusted to fit the Sandage-Tammam[29] velocity data.

Peeble defines a function of Ω as

$$f(\Omega) = \left[\frac{d \ln c(t)}{d \ln a(t)}\right]_{t=t_o} \tag{41}$$

where a(t) is the expansion parameter and c(t) is a linear density contrast function defining the relative increase in density of the supercluster over the background. More directly $f(\Omega) = c^{-1} \frac{(dc/dt)}{(H)}$, thus a measure of the logarithmic growth of the density over the expansion of the model. The peculiar velocity field is given by

$$V = \frac{HAf(\Omega)}{3-\gamma} \frac{1}{d^{\gamma-1}} = \frac{c}{d^{\gamma-1}} \tag{42}$$

Combining equations (40) and (41) $N(d) = \langle N\rangle \left[1 + 80/d^2 f(\Omega)\right]$.

Now comparing this to an Abell cluster mean density, Peebles finds

$$N_a(d) \simeq N \quad 1 + (700/d^2) \tag{43}$$

Because the virgo cluster is considered somewhat less massive than an Abell cluster, a Ω of 1.0 or 0.1 would give a reasonable value here. For the quantitative side, the luminosity function of Shapero[30] is well approximated by the Abell form

$$N(M) = Kdex\,(1.33M), \quad M<M^* \tag{44a}$$

$$Kdex\,(0.33M), \quad M\geq M^* \tag{44b}$$

$$M^* \simeq -19.0 + 5 \log h \tag{44c}$$

Now h = 0.57 from the data of Sandage and Tammann.[24] So that $M^* = -20.2$ and the distance of a galaxy with absolute magnitude M^* and seen at apparent magnitude $M_o = 13$ (cutoff magnitude from galaxy count[30]) is

$$D^*(M_o = 13) \simeq 43\text{Mpc} \tag{45}$$

so that the angular distribution of galaxies for $M<M_o$ (number per steradian) is related to space density by

$$N(\Theta) = \langle N\rangle \int_o^\infty r^2 dr \phi(r/D^*)(1 + (A/d^r)) \tag{46}$$

where Θ is the angular distance between the virgo cluster and the observed galaxy from our view point and r is the proper distance of the galaxy from us and d is the distance from the observed galaxy to the center of the virgo cluster, and

$$(r/D) = N(<M=M^*-5\log(r?D)) \quad (47)$$

Equation (46) can be rewritten

$$N(\Theta) = \left[1 + (A/D^{*\gamma})I(\Theta,M_o)\right] \cdot N \quad (48a)$$

where

$$I(\Theta,M_o) = \left[\int_0^\infty S^{-\gamma}x^2dx\phi(x)\right] / \left[\int_0^\infty x^2dx\phi(x)\right], \quad (48b)$$

and where

$$s^2 = x^2 + (Y/D^*)^2 - 2x(Y/D^*)\cos\Theta \quad (48c)$$

where

$$N = \langle N \rangle D^{*3}\int_0^\infty x^2dx\phi(x) \quad (49)$$

is the mean number density of galaxies brighter than M_o, the limiting magnitude. Combining equations (41) and (47) gives

$$\begin{aligned} N_i &= \alpha\Delta\Omega_i\left[1 + BI_i(M_o)\right] \\ B &= \left[(3-\gamma)c\right] / \left[HD^{*\gamma}f(\Omega)\right] \end{aligned} \quad (50)$$

where α is a scale factor and $\Delta\Omega_i$ is the solid angle subtended by the ith angle bin at galactitic latitude $/b^{II}/$ 40°, I_i is the integral in equation (48), and C recalls equation (42). By comparing the values $\Omega = 1.0$ and $\Omega = 0.1$ in these calculations to the observational results of de Vaucoulers and de Vaucoulers[31] it is seen that neither $\Omega = 0.1$ or $\Omega = 1.0$ is out of order although the data are closer to $\Omega = 0.1$. Therefore $\Omega \leq 1$ is not unreasonable and the universe is at least flat and probably open.

N-Body Simulations of Galaxy Clustering

The last model-based method we consider is a mathematically simulated cosmology, a computer N-body simulation.[32] The simulation had two objectives. The first one was to improve present understanding of the galaxy clustering process and the second objective was to obtain a value of Ω for a model that fit present day observations.[33]

It was found that these observations fit a simulated model with Ω equal to 0.1 best and not a model that had the value Ω equal to 1.0.[33]

The authors simulate a galaxy cluster as a spherical region of the universe of radius R expanding with velocity $\dot{R}$ centered on a given origin, containing 1000 mass points with positions X_i, Y_i, Z_i, and velocities $\dot{X}_i$, $\dot{Y}_i$, and $\dot{Z}_i$. In the above model, simulated coordinates analogus to right ascension Θ_i, declinations ϕ_i, and radial velocities V_i of the sample galaxies are defined

$$\Theta_i = \arctan(Y_i/Z_i) \tag{51}$$

$$\phi_i = \arctan\,(Y_i^{\,2} + Z_i^{\,2})^{\frac{1}{2}}/(R - X_i) \tag{52}$$

$$V_i = (\dot{R} - \dot{X}_i)(R - X_i) + Y_i\dot{Y}_i + Z_i\dot{Z}_i/\ (R-X_i)^2 + Y_i^{\,2} + Z_i^{\,2\ \frac{1}{2}} \tag{53}$$

These parameters represent the simulation as it would appear to an observer on the edge of the sphere at X = R and Y = Z = 0. The area in the short line of sight to the observer is ignored, i.e., those observations for which $\phi > 45^{\circ}$. This accounts for about one quarter of the volume of the sphere.[32]

In the real universe, galaxies have a broad distribution of luminosities; at best, a given sample is magnitude-limited. The simulated sample is distance-limited, and all simulated galaxies are taken to have the same luminosity. For uniformly distributed galaxies in a magnitude-limited sample with a given luminosity function, the differential probability distribution of galaxies at distance r is given by

$$P(r)dr = (3/\Gamma(3/2)r^{*3})r^2E_i(r^2/r^{*2})dr \tag{54}$$

where $E_i(x) = \int_x^{\infty} t^{-1}e^{-z}dt$ is the exponential integral and r* is the distance at which a galaxy of luminosity L* would have the limiting apparent magnitude of the sample, and Γ is the Γ - function of probability.

One of the most striking differences in the two models simulated by the authors is the difference in the distribution of the group masses given by the simple integrated count distribution of group masses $f_g(N)$ which is defined by

$$f_g(N) = \frac{1}{N_T} \sum_{i=1}^{N} i\, M_g(i) \qquad (55)$$

where N_T is the total number of galaxies in the sample and $M_g(M)$ is the number of groups with M members.

This distribution gives a much larger spread in group richness for the model with $\Omega = 0.1$ which fits present observations better than the distribution that occurs with the model for $\Omega = 1$.[32,33] In the above models, each mass point represents a galaxy of luminosity L^*_{pg} ($3.4 \times 10^{10} L_\odot$ and mass $5 \times 10^{13} \Omega M_\odot$). The above simulation concludes that the mean mass to light ratio of groups of galaxies[20] is within a factor of 2 of 100 $(M_\odot/L_\odot)$ and therefore that distributions with galaxies contributes $\sim$10% of the crucial density required to close the universe; i.e., $\Omega = 0.1$.[32,33] The simulated models give observational pictures which fit published data best for $\Omega = 0.1$ models, and not the models for which $\Omega = 1.0$.

Spectrophotometry and the Hubble Diagram

With the advent of the polamar multichannel spectrometer and its ability to subtract the sky background accurately, an approach to cosmology using spectrophotometry of faint cluster galaxies to construct the Hubble diagram became possible.[34] After doing the appropriate spectrophotometric observations over a wide frequency range of faint galactic clusters, one may construct a Hubble redshift magnitude diagram just as has been done previously with other data.[35,36]

The monochromatic flux F_ν from a source of luminosity L_ν at coordinate distance X is

$$F_v = \frac{L_v(1 + Z)}{4\pi R_R{}^2 \Sigma^2(X)(1 + Z)} \tag{56}$$

where R_R is the scale factor when the radiation is received and $(1 + Z) = (R_R/R_E)$ where R_E is R at the epoch of emission. For the case where the cosmological constant and the pressure both vanish, the model is uniquely specified by H_o, the present Hubble constant, and q_o, which were defined in equations (17) and (18) as $H_o = (\dot{R}/R)_o$ and $q_o = (\ddot{R}R/\dot{R}^2)|_o = (4\pi G\rho_o)/(3H_o{}^2)$ where ρ_o is the present mean matter density. Coordinates can be chosen such that the scale factor R is proportional to (C/H_o), the Hubble distance, and the quantity $\Sigma(X)$ is a function of q_o and Z only. Let the luminosity distance Lq(Z) be given by

$$Lq(Z) = H_o R_o \Sigma q_o \left[X(Z_1, q_o)\right] C^{-1} \tag{57}$$

where $Lq \simeq Z$ for small Z. Then

$$Lq(Z) = \frac{q_o Z + (q_o - 1)\left[(1 + 2q_o Z)^{\frac{1}{2}} - 1\right]}{q_o{}^2(1 + Z)} \tag{58}$$

and

$$F_v = \frac{H_o{}^2 L_v(1 + Z)}{4\pi C^2 Lq^2(Z)(1 + Z)} \tag{59}$$

For the angular diapham used in the study $q_o = \frac{1}{2}$ implies a standard distance $r_o = 16$ Kpc at the redshift of the source.[34] Thus the diaphragm radius is

$$\gamma = \frac{r_o}{R\Sigma(X)} = \frac{H_o r_o(1 + Z)}{CLq(Z)} \tag{60}$$

The real projected radius if

$$r = R\Sigma(X)\gamma = r_o \frac{L_q(Z)}{L_{\frac{1}{2}}(Z)} \tag{61}$$

and the observed flux is

$$F_v = \frac{H_o{}^2 L_{ov}(1 + Z)}{4\pi C^2(1 + Z)Lq^{2-\alpha}(Z)L_{\frac{1}{2}}{}^{\alpha}(Z)} \tag{62}$$

Where the fact that $L_v = L_{ov}(r/r_o)^{\alpha}$ gives the luminosity as a power of the projected diaphragm radius.

For the construction of the Hubble diagram, the authors define

ζ the distance modulus, to an additive constant or just "distance" and the magnitude S, for $q_o = \frac{1}{2}$ as

$$\zeta = 5 \log (1+Z)L_{\frac{1}{2}}(Z) + k_s(Z) \tag{63}$$

$$S = 2.5 \log(1 = Z) + V_I + k_s(Z) \tag{64}$$

where V_I is the approximate intrinsic magnitude and $k_s(Z)$ is a smoothed version of the K-correction and is equal to $0.918 \tan^{-1} 5.20(Z - 0.340) + 0.970$, (the accuracy of the adopted k-corrector is irrelevant except for its effect on the statistics)[34], also they use the definitions

$$S = \mu + \zeta \tag{65}$$

and

$$\mu = M_{V_I} + 5 \log(c/H_o) - 5 \tag{66}$$

where M_{V_I} is the absolute monochromatic magnitude at $\log V_o = 14.740$. μ is called the reduced absolute magnitude. The magnitude S for other values of q_o is

$$\begin{aligned} S &= \mu + \zeta + 2.5(2 - \alpha) \log Lq(Z)/L_{\frac{1}{2}}(Z) \\ &\equiv \mu + fa(\xi) \end{aligned} \tag{67}$$

where ζ is the distance or distance modulus and μ is the reduced absolute magnitude of equation (66). The distribution function ζ at a given S is

$$\zeta \equiv \zeta_* + \xi\sigma^2 \tag{68}$$

where ζ is the distance a source of absolute magnitude μ_o, observed at an apparent magnitude S, would have if $q = q_o$ and a correction factor ξ is

$$\xi = \left(\frac{dfq}{dZ}\right)^{-2} \left(\frac{2\,\frac{dLq}{dZ}}{Lq} - \frac{1}{1 = Z} - \frac{q_o}{1 = q_o t} - \frac{\frac{d^2fq}{dZ^2}}{dfq/dZ} \right) \frac{d\zeta}{dZ} \tag{69}$$

and $d\zeta/dfq \gtrless 1$. The authors found from their analysis a likely value of q_o to be less than (0). The optical properties are similar to those for small redshifts or K = -1. This seems to support an open universe. However, it was pointed out by Tinsley[37] that evolutionary effects on the data may cause large changes. The authors point out that more research needs to be done in this area.

Unbound Universe

We draw attention to the compendium of research by Gott, Gunn, Schramn, and Tinsley.[38] The Hubble parameter in their analysis varies between 30 and 120 $Kmsec^{-1}Mpc^{-1}$. The age of the universe t_o is given by

$$t_o = f(\Omega)/H_o \tag{70}$$

where $f(\Omega) = \frac{\Omega}{2}(\Omega - 1)^{3/2}\mathrm{eos}^{-1}(\frac{1}{\Omega} - 1) - \frac{1}{\Omega}(\Omega - 1)^{1/2}$

for $\Omega > 1$, $f(\Omega) = 2/3$ for $\Omega = 1$ and for $\Omega < 1$,

$f(\Omega) = (1 - \Omega)^{-1} - \Omega/2(1 - \Omega)^{-3/2}\cosh^{-1}(\frac{2}{\Omega} - 1)$.

if $\Omega = 0$ then $f(\Omega) = 1 + \frac{\Omega}{2}\Big|_n \Omega$. The lower and upper limits of t_o are determined to be 8 and 18 billion years respectively. From the values of redshifts of galaxies (Hubble Diagram) the authors determined that the upper limits q_o and Ω are less than 2 and 4 respectively. Their analysis of the uniformity of expansion or deviation from the Hubble expansion due to density perturbations led to a value of Ω less than one. Through the analysis of redshifts and magnitudes of individual galaxies, nearer clusters of galaxies, rich clusters, and the virial theorem for our galaxy by techniques described in previous sections of this thesis, they place a best lower limit on the value of Ω^* (i.e. the mass contribution of galaxies alone to the total mass density of the universe) at about 0.05, rather low.

The origin of galactic interstellar deuterium is also discussed. The direct spectroscopic measure of deuterium in interstellar space gives a number ratio of $D/H = 1.4 \times 10^{-5}$ or a mass fraction X_D of

2.0×10^{-5}. The mass fraction is strongly related to the mean density of matter ρ_o where $\rho_o = \frac{3H_o^2}{4\pi G}qo$.

In considering element production it is necessary to consider only what occurs after the temperature has dropped below $\sim 10^{11}$degK, since strong and weak electromagnetic interactions are strong enough to keep all particles in statistical equilibrium above this temperature.[39] In addition, the photon flux prevents the neutrons and protons from combining until the photons have been cooled by the expansion to $\sim 10^9$degK, at which time nucleosynthesis can commence.[40]

The work-energy relation for a gas total mass-energy density ρ and pressure P can be written

$$\frac{d}{dV}(\rho V) + \frac{P}{C^2} = 0 \tag{71}$$

where V is an element volume as measured by an observer moving with the matter[39] (this is assuming homogeneity) and serves to relate temperature T to volume once $\rho(t)$ and $P(t)$ are specified. Temperature T is the thermal equilibrium temperature between electrons, baryons, and photons until the plasma recombines at $\sim 10^9$degK.[40] Expansion rates that are very slow produce few nuclei, since many of the neutrons have time to decay before element synthesis begins. As the expansion rate increases, production rises due to the increased availability of neutrons. With larger expansion rates even deuterium creation stops.[39] Therefore the mean density and expansion rate are closely related to the mass fraction of deuterium. For a realistic estimate the authors chose the deuterium fraction $X_D = 2.0 \times 10^{-5}$ to be half the primordial value. To synthesis $X_D = 4 \times 10^{-5}$ requires ρ_o, to equal $4 \times 10^{-31} g/cm^3$. It is shown that this low density is consistent with the value of Ω^* as the lower limit of Ω and that combined with the upper age limit of the universe exerts constraints such that Ω and H_o should range 0.09 to 0.05 and 49 to 65 km/sec Mpc^{-1} respectively. Using the most minimal estimates of X_D and ρ_o, the constraints range 0.05 to 0.2 and 47 to 120 km/sec Mpc^{-1} respectively. There are arguments that deuterium can be produced in large shock-wave envelopes of massive stars and supernova.[40] The authors studied the production of boron and beryllium in these shock waves because the energy per nuclear of deuterium in a shock is not well known. The ratios, B/D and Be/D, are almost independant of shock strengh and are much greater than the observed abundance ratios, which means that even if all the observed B and Be are produced in S-N shock, the amount of D produced is still much less than that observed. Also deuterium is destroyed by astration more readily than B or Be, so the discrepancy is enhanced by galactic evolution. Although there are many loopholes, all of the other strongest arguments taken together point to an open universe with $\Omega \simeq 0.06$. The data from deuterium production also point toward a value of $\Omega \simeq 0.06$. It is possible to construct an open model where (1) the deuterium production is consistent with observations, (2) the mass density of the universe exceeds that known to be in galaxies, and (3) the age of the universe is consistent with the age of the elements and the globular clusters. Satisfaction of these constraints limits Ω to certain values (i.e., $0.05 < \Omega < 0.09$) and H_o to a small range (i.e., $49 < H_o < 65$). These constraints imply an open universe, thus indicating an open model of the universe, by which present day observations can be explained, is highly probable.[38]

Anisotropy of the Universe

When isotropy (or uniformity of observation in all directions) is discussed, the most important datum in existence is the microwave background radiation.[41] Its remarkable degree of isotropy implies a high degree of uniformity of the universe back to a redshift of at least Z ≈ 1000. More recently refernces have been made to elemental abundance observations. The expansion rate of the universe is affected by the amount of anisotropy present; the same expansion rate affects the amount of heavy elements produced, as noted in an earlier section of this thesis.[38] One further datum is the anisotropy of mass distributions in observations which would have much information about the microwave background (i.e., gravitational shifts of frequency along the world-path of a photon.)[42]

Much of the discussion of isotropy or anisotropy is dedicated to the microwave background blackbody radiation, which presently has a temperature of about 3 degK.[43] This background radiation is considered to be the remnant radiation of the primordial fire-ball of the big-bang theory. This microwave background radiation has always been thought to be highly isotropic and generally is.[44] Silk has hown that the detection of scale angular variations in the microwave background radiations will provide a direct means of ultimately vertifying the most viable and generally accepted class of current theories of galaxy formation.[45] In 1977 Snort, Gorenstein, and Muller found an anisotropy in the microwave background on the order of 1 in 3000. A very small ratio indeed but it is still significant. The fact that the universe is extremely isotropic now doesn't mean it was always so to such a degree. Homogeneous and isotropic configurations are not likely to have occured in the early stages of the universe, because of the light curve structure of the Friedman models and the instability of such isotropic spaces under perturbations near the singularity.[44] Anisotropic models evolve toward isotropic configurations during the radiation-dominated era, but a resisual amount of anisotropy is expected to remain in the background.[44] With the resisual amount of anisotropy in mind we will assume that the early universe was anisotropic and will use that as the basis of the model. We also point out that Snort, Gorenstein, and Muller could have been incorrect in their analysis of the dipole anisotropy; (i.e., the anisotropy was due to the earth moving against the rest frame of the cosmic background radiation.) There are anisotropic models with dipole anisotropies.[4]

Although it is possible that their analysis is correct, it is also possible that the anisotropy that the anisotropy that was detected is the remnant of larger anisotropies of the early history of the universe predicted previously.[44] The most important mechanism in reducing the larger anisotropies of the past is neutrino viscosity at temperature above 10^{10}degK, when the freq..ency for collusions between neutrinos and thermal electrons or positrons is comparable to the expansion rate. Further reductions in anisotropy take place during the radiation dominated expansion phase. When the temperature is between 10^{10}degK and 10^{7}degK, neutrinos are collisionless; and the anisotropic stresses from the anisotropic momentum distribution in the neutrino radiation must be taken into account.[43]

CHAPTER III. FORMALISM

Anisotropic Cosmological Models

In the remainder of this paper, ' · ' indicates the time derivative; ( ) and [ ] indicate the symetric and antisymetric parts of the decomposition of a tensor. In recent years anisotropic cosmologies have been studied by many researches. Since the advent of Misner's benchmark paper on anisotropic Bianchi I cosmology in 1968,[43] there have been many papers on this topic. Among these are studies of Bianchi V and X by Matzner [46,47] and by Hawking;[48] I, V, VII, and IX by Collin and Hawking;[14] X by Matzner, Shepley and Warren;[49] all types without fluid flow by Ellis and MacCallum;[50] all perfect fluid Bianchi type with flow by King and Ellis;[4] and many others too numerous to list. With these studies as our guide, we proceed to outline our formalism, drawing mainly on Collins and Hawking:[48]

One may define three invariant Vector fields $E_A{}^\mu$ in the surfaces of homogeneity which are dual to the $E^A{}_\mu$

$$E^A{}_\mu E_B{}^\mu = \delta_\beta{}^A \qquad (72)$$

then

$$E^A{}_\mu E_A{}^\upsilon = \delta_\mu{}^\upsilon + n^\upsilon n_\mu \qquad (73)$$

where $N_\mu = -t_{//\mu}$ is the normal to the surfaces of homogenity. Any tensor can be expressed in terms of its components with respect to the $E^A{}_\mu$, N_μ and N^μ. If the field is invariant under a group of isometries, the components will be functions of time only. The fluid flow-vector of matter can be expressed as

$$u^{\mu} = u^{o}n^{\mu} + u^{A}E_{A}{}^{\mu} \tag{74}$$

where $u^{o} = -u^{\mu}n_{\mu}$ and $u^{A} = u^{\upsilon}E^{A}{}_{\upsilon}$. Capitol Latin indices may be raised and lowered by the matrix g_{AB} and its inverse g^{AB} (i.e., $g_{AB} = g_{\mu\upsilon}E_{A}{}^{\mu}E_{\beta}{}^{\upsilon}$ and $g^{AB} = g^{\mu\upsilon}E^{A}{}_{\mu}E^{\beta}{}_{\upsilon}$).

The matrix may be split into its volume and distortion parts

$$g_{AB} = e^{2\alpha}(e^{2\beta})_{AB} \tag{75}$$

where α and β_{ij} are functions only of time; β_{ij} is a symetrical trace-free 3 X 3 matrix and $e^{2\beta}{}_{ij}$ is the series $\overset{\infty}{\Sigma}(r!)^{-1}(2\beta)^{r}{}_{ij}$. An orthonormal basis X^{γ}_{μ} can be defined where $X^{o}{}_{\mu} = -n_{\mu}$ and

$$X^{i}{}_{\mu} = e^{-\alpha}(e^{-\beta})_{iA}E^{A}{}_{\mu}. \tag{76}$$

We can now define the Ricci rotation coefficients

$$X^{\gamma}_{\mu}||_{\upsilon} = \Gamma^{\ \ \upsilon}_{\delta E}X^{\delta}_{\mu}X^{\varepsilon}_{\upsilon}, \tag{77}$$

giving the variations in X^{γ}_{μ} as it is dragged in a Fermi-transported frame through spacetime, where $\Gamma_{\delta\varepsilon\gamma} = -\Gamma_{\gamma\varepsilon\delta}$. This gives that

$$\Gamma_{\gamma\delta\varepsilon} = X_{\gamma[\mu||\upsilon]}X_{\delta}{}^{\mu}X_{\varepsilon}{}^{\upsilon} + X_{\varepsilon[\mu||\upsilon]}X_{\gamma}{}^{\mu}X_{\delta}{}^{\upsilon} - X_{\delta[\mu||\upsilon]}X_{\varepsilon}{}^{\mu}X_{\gamma}{}^{\upsilon} \tag{78}$$

and

$$E^{A}{}_{\mu||\upsilon} - E^{A}{}_{\upsilon||\mu} = C_{BC}{}^{A}E^{B}{}_{\mu}E^{C}{}_{\upsilon} \tag{79}$$

where $C_{BC}{}^{A} = \varepsilon_{BCD}N^{AD} + a_{B}\delta_{C}{}^{A} - a_{\varepsilon}\delta_{B}^{A}$ and ε_{BCD} is the permutation operator, and the tensors N^{AD} and a_{B} are relative tensors in the three-dimensional space. Equation (79) defines an algebra of which the $C_{BC}{}^{A}$ are the structure functions. The information contained in Equations (78) and (79) may then be used to find an explicit form of the rotation coefficients

$$\Gamma_{ijo} = -\Gamma_{oji} = \dot{\alpha}\delta_{ij} + \alpha_{ij} \tag{80a}$$

$$\Gamma_{ioo} = -\Gamma_{ooi} = 0 \tag{80b}$$

$$\Gamma_{ioj} = -\tau_{ij} \tag{80c}$$

$$\Gamma_{ijk} = \tfrac{1}{2}e^{-\alpha}(e^{\beta})_{FA}(e^{-\beta})_{GB}(e^{-\beta})_{HC}C_{BC}{}^{A}$$
$$\left[\delta_{Fi}(\delta_{Gi}\delta_{Hk}) + \delta_{Fk}\delta_{Gi}\delta_{Hj} - \delta_{Fj}\delta_{Gk}\delta_{Hi}\right] \tag{80d}$$

where $\sigma_{ij} = (\dot{e}^{\beta})_{k(j}(e^{-\beta})_{i)k}$ represents the shear of the normals n_u and $\tau_{ij} = (\dot{e}^{\beta})_{k\ i}(e^{-\beta})_{j\ k}$ represents the extent to which $(\dot{e}^{\beta})$ does not commute with e^{β}, which gives a measure of how the normals n_u rotate with respect to a frame fixed in the spatial hypersurfaces. In spacetime the Riemann curvature tensor is defined by $\nabla_a U_b - \nabla_b U_a = R_{cdba}U^c$ where u^a is any vector and ∇_a is the covariant derivative. The Ricci tensor is then defined by contracting $R_{ca} = R_{cdab}$. In the orthonormal basis the components of the Ricci tensor are

$$R_o{}^o = 3\ddot{\alpha} + 3\dot{\alpha} + \sigma_{ij}\sigma_{ij} \tag{81a}$$

$$R_i{}^o = e^{-\alpha}\left[(e^{-\beta}\sigma e^{+\beta})_{\beta A}C_{BC}{}^{A}(e^{-\beta})_{Ci} - \sigma_{ij}(e^{-\beta})_{jC}C_{AC}{}^{A}\right] \tag{81b}$$

$$R_{ij} = R^*_{ij}\left[\ddot{\alpha} + 3\dot{\alpha}^2\right]\delta_{ij} + (\dot{\sigma}_{ij} + 3\dot{\alpha}\sigma_{ij} + \sigma_{ik}\tau_{kj} - \tau_{ik}\sigma_{kj}) \tag{81c}$$

where R^*_{ij} is the Ricci tensor of the surfaces of homogeneity:

$$R^*_{ij} = -\tfrac{1}{4}e^{-2\alpha}\{2C_{BC}{}^{A}C_{DA}{}^{C}(e^{-\beta})_{Di}(e^{-\beta})_{Dj}$$
$$+C_{BC}{}^{A}C_{DE}{}^{C}(e^{-2\beta})_{BE}\left[2(e^{2\beta})_{DA}(e^{-\beta})_{Ci}(e^{-\beta})_{Fj} - (e^{-2\beta})_{CF}(e^{\beta})_{Ai}(e^{\beta})_{Dj}\right]$$
$$+ 2C_{AB}{}^{A}C_{DE}{}^{C}(e^{-2\beta})_{BE}\left[(e^{\beta})_{Ci}(e^{\beta})_{Dj} + (e^{\beta})_{Cj}(e^{-\beta})_{Di}\right]\} \tag{82}$$

The curvature scalor is

$$R = 6\ddot{\alpha} + 12(\dot{\alpha})^2 + \sigma_{ij}\sigma_{ij} + R^* \tag{83}$$

The Einstein tensor is then $G_{AB} = R_{AB} - \tfrac{1}{2}g_{AB}R$ and the field equations are $G_{ab} = -8\pi T_{ab}$ when T_{ab} is the energy-momentum tensor. This leaves the field equations to be

$$3\dot{\alpha}^2 - \tfrac{1}{2}\sigma_{ij}\sigma_{ij} + \tfrac{1}{2}R^* = 8\pi T^{00} \tag{84a}$$

which gives the inertia density,

$$e^{-\alpha}\left[(e^{-\beta}\sigma e^{\beta})_{BA}C_{BC}{}^{A}(e^{-\beta})_{Ci} - \sigma_{ij}(e^{-\beta})_{jC}C_{AC}{}^{A}\right] = 8\pi T^{oi} \tag{84b}$$

which gives the momentum density,

$$\dot{\sigma}_{ij} + 3\dot{\alpha}\sigma_{ij} + \sigma_{ik}\tau_{kj} - \tau_{kj}\sigma_{kj} + R^*_{ij} - \frac{1}{3}R^*\delta_{ij} = 8\pi\left[T_{ij} - \frac{1}{3}T_{kk}\delta_{ij}\right] \tag{84c}$$

which gives the trace-free anisotropic stresses, and

$$-6\delta - 9(\dot{\alpha})^2 - \frac{3}{2}\sigma_{ij}\sigma_{ij} - \tfrac{1}{2}R^* = 8\pi T_{kk} \tag{84d}$$

which gives the scalor (traces of T^k_k) isotropic pressure.

Geodesics

The parallely transported target vector L^μ to a geodesic obeys $L^\mu{}_{||\upsilon}L^\upsilon = 0$. In components with respect to the orthonormal basis this gives[14,48]

$$L^o\dot{L}^i + \Gamma^i_{jk}L^iL^k + \Gamma^i_{oj}L^oL^j + \Gamma^i_{jo}L^jL^o = 0 \tag{85}$$

where $L^o = (L^iL^i)^{\frac{1}{2}}$ for null geodesic and $L^o = (1 + L^iL^i)^{\frac{1}{2}}$ for timelike geodesics. Equation (85) has a simple form in terms of components with repsect to the $E^A{}_\mu$

$$\dot{L}_A = (L^o)^{-1}C_{CA}{}^BL_BL_De^{-2\alpha}(e^{-2\beta})_{CD}. \tag{86}$$

L_A is nearly constant for a time-like geodesic for which $(L^iL^i)^{\frac{1}{2}}$ is small.

Observations

The background radiation can be considered, to a first order approximation, as coming from a surface of homogeneity in the past corresponding to the last time the radiation was scattered. The received temperature, T, in a given direction will be $T_R = T_E(1 + Z)^{-1}$ where T_E is the temperature of the emitter and Z is its redshift in that direction, which is given by[14,48]

$$(1 + Z) = \frac{U_E{}^\mu K_\mu}{U_R{}^\mu K_\mu} \tag{87}$$

where $U_R{}^\mu$ is the velocity vector of the receiver, $U_E{}^\mu$ is the velocity vector of the matter at the emitting surface, and K^μ is the target vector to the null geodesic from the receiver in a given direction. Now in general

$$T_R = T_E(U_R{}^o + K_R{}^iU_R{}^i)\left[(K_E{}^jK_E{}^j)^{\frac{1}{2}}N_E{}^o + K_E{}^iU_E{}^i\right]^{-1} \tag{88}$$

where $K_R{}^o$ is taken to be minus one and the term $(K_E{}^iK_E{}^i)$ gives the redshift resulting from the expansion of the Universe. The time $K_R{}^i U_R{}^i$ gives the dipole variation from the present peculiar velocity of the emitter. Expansion of the equation (88) to first order in the kinematical quantities is sufficient because the present microwave background measurements cannot measure higher harmonics (second order brings in octupole terms). This gives

$$T_R = T_E e^{\alpha E - \alpha R}(1 + K_R{}^i U_{Ri} - K_E{}^i U_{Ei} - \int_E^R K^i K^j \sigma_{ij} dt) \qquad (89)$$

where the integral gives the quadrupole shear terms (12-hour variations). The $K^i U_i$ terms give the dipole (24-hour) variations. We use the normal shear σ_{ij} instead of the true fluid shear.

Fluid Kinematics

The gradients of the fluid velocity U_μ can be expressed in terms of the expansion Θ, the shear Σ, the vorticity $\omega_{\mu\upsilon}$, and the acceration A_μ of the flow congruence as

$$U_{\mu||\upsilon} = \Sigma_{\mu\upsilon} + \frac{1}{3}\Theta\delta_{\mu\upsilon} + \omega_{\mu\upsilon} - A_\mu U_\upsilon \qquad (90)$$

where $\sigma_{\mu\upsilon}U^\upsilon = \omega_{\mu\upsilon}U^\upsilon = 0$, $\Sigma_\mu{}^\upsilon = 0$ and $\Sigma_{(\mu\upsilon)} = \omega_{(\mu\upsilon)} = 0$. The separate parts of the gradient are defined by

$$\Sigma_{\mu\upsilon} = \Theta_{\mu\upsilon} - \frac{1}{3}\delta_{\mu\upsilon}\Theta + A_{(\mu}U_{\upsilon)} \qquad (91a)$$

$$\Theta_{\mu\upsilon} = U_{(\mu||\upsilon)} \qquad (91b)$$

$$\omega_{\mu\upsilon} = U_{[\mu||\upsilon]} + A_{[\mu}U_{\upsilon]} \qquad (91c)$$

$$A_\mu = U_{\mu||\upsilon}U^\upsilon \qquad (91d)$$

where for any tensor $C_{\alpha\beta}$, $C_{(\alpha\beta)} = \frac{1}{2}(C_{\alpha\beta} + C_{\beta\alpha})$ and $C_{[\alpha\beta]} = \frac{1}{2}(C_{\alpha\beta} - C_{\beta\alpha})$, and $h_{\mu\upsilon} = g_{\mu\upsilon} + U_\mu U_\upsilon$ is the projection operator in the observer rest space orthogonal to the flow vector U_μ. We normalize U_μ such that $U_\mu = -1$.

Energy-Momentum Tensor

The energy momentum tensor for the matter will be given by that for a viscous fluid

$$T^{m}_{\mu\upsilon} = \mu U_{\mu} U_{\upsilon} + h_{\mu\upsilon} p + \pi_{\mu\upsilon} \tag{92}$$

where μ is the inertia density, p is the isotropic pressure, and $\pi_{\mu\upsilon}$ are the anisotropic viscous stresses. We will use as equation of state $\mu = (\gamma - 1)p$ where the speed of sound a_s is given by $a_s = (\gamma - 1)^{\frac{1}{2}}$. The viscous stresses are given by $\pi_{\mu\upsilon} = -\lambda\Sigma_{\mu\upsilon}$ where λ is the kinematic viscousity.

The electromagnetic field will be defined by

$$F_{\mu\upsilon} = -U_{[\mu}E_{\upsilon]} + \tfrac{1}{2}\eta_{\mu\upsilon\gamma\delta}U^{\gamma}H^{\delta} \tag{93}$$

where E_{υ} and H^{γ} are the electric and magnetic fields and $\eta_{\mu\upsilon\gamma\delta}$ is the four-dimensional permutation symbol. The energy momentum tensor for the electromagnetic field is

$$T^{EM}_{\mu\upsilon} = F_{\mu\alpha}F_{\upsilon}{}^{\alpha} - \tfrac{1}{4}g_{\mu\upsilon}F_{\alpha\beta}F^{\alpha\beta} \tag{94}$$

If we decompose $T_{\mu\upsilon}$ according to our representations in the Einstein equations we get

$$T_{oo} = (\mu + p)U_o{}^2 - p + E^2 + H^2 \tag{95a}$$

$$T_{oi} = (\mu + p)U_o U_i + U_o \eta_{ijk} E^j H^k \tag{95b}$$

$$T^k{}_k = 3p + E^2 + H^2 \tag{95c}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T^k{}_k =$$

$$(\mu + p)\left[U_i U_j - \frac{1}{3}\delta_{ij}U_k U^k\right] + \frac{1}{3}\left[E^2 + H^2\right] - E_i E_j - H_i H_j - \lambda\Sigma_{ij} \tag{95d}$$

<u>Magnetohydrodynamics</u>

The problem of this thesis is a Magnetohydrodynamic (MHD) cosmology. MHD cosmologies have recently been considered by Tupper,[51] in Bianchi type-I. Dunn and Tupper have studied MHD Bianchi type VI cosmology with[52] and without[53] fluid flows. Tupper has shown how such cosmologies can constrain the conductivity.[54]

Maxwell's equations will be used in the form

$$F_{\mu\upsilon}{}^{||\upsilon} = J_{\mu} \tag{96a}$$

$$\eta^{\mu\upsilon\lambda\rho}F_{\upsilon\lambda||\rho} = 0 \tag{96b}$$

where the current $J_\mu = \rho U_\mu - \eta F_{\mu\upsilon}U^\upsilon$ (96c)

where ρ is the free charge density and η is the ohmic conductivity.

The energy momentum conservation law is

$$T^{\mu\upsilon}{}_{||\upsilon} = 0 \tag{97}$$

from which we take the two natural projections

$$U_\mu T^{\mu\upsilon}{}_{||\upsilon} = 0 \tag{98a}$$

$$h^\lambda_\mu T^{\mu\upsilon}{}_{||\upsilon} = 0 \tag{98b}$$

Using these and Maxwell's equations we find

$$\dot{\mu} + (U + p)\Theta = -\pi_{ij}\Sigma^{ij} - J_i E^i \tag{99a}$$

and

$$(\mu + p)A_i = -h^j_i(\nabla_j p + \nabla_k \pi_j{}^k) - \eta_{jk\ell}J^k B^\ell \tag{99b}$$

where $B^\ell = H^\ell/M$ with M the magnetic permitivity which we henceforth set M = 1. For MHD conditions to hold we must have

(1) $\eta \to \infty$ (100a)

(2) $\rho \to 0$ (100b)

(3) $E_i \to \eta_{ijk}U^j H^k$ (100c)

(4) $J_o = 0$ (100d)

In reality η is finite so we will use equation (96c) for J_μ subject to restrictions (2), (3), and (4) above.

CHAPTER IV. A BIANCHI V MHD COSMOLOGY

The Model

We now examine a Bianchi type V anisotropic cosmology. Type V cosmologies with general fluids have been studied by Matzner,[47] Hawking,[48] Collins and Hawking,[14] and Batakis and Cohen.[48] For simplicity we choose β_{ij} diagonal. The metric is then

$$ds^2 = -dt^2 + e^{2\alpha}(e^{2\beta_1}dx^2 + e^{2(\beta_2 + \chi)}dy^2 + e^{2(\beta_3 + \chi)}dz^2) \tag{101}$$

This means that

$$C^2{}_{[21]} = C^3{}_{[31]} = 1 \tag{102}$$

and the rotation coefficients are[47,48]

$$\Gamma^1_{22} = -\Gamma^2_{21} = \Gamma^2_{12} = -2e^{-\alpha}e^{-\beta_{11}} \tag{103a}$$

$$\Gamma^1_{33} = -\Gamma^3_{31} = \Gamma^3_{13} = -2e^{-\alpha-\beta_{11}} \tag{103b}$$

$$\Gamma_{ijo} = -\Gamma_{oji} = \dot{\alpha}\delta_{ij} + \sigma_{ij} \tag{103c}$$

$$\Gamma_{ioj} = \Gamma_{ioo} = 0 \tag{103d}$$

where $\sigma_{ij} = \dot{\beta}_{ij}$

Then

$$R^*{}_{ik} = \frac{1}{3}\delta_{ik}R^* \tag{104a}$$

with

$$R^* = -6e^{-2\alpha}e^{-2\beta_1} \tag{104b}$$

The field equations are

$$3\dot{\alpha} - \tfrac{1}{2}\sigma_{ij}\sigma^{ij} - 3e^{-2\alpha}e^{-2\beta_{11}} = 8\pi T^{oo} \tag{105a}$$

$$G_{oi} = 0 \tag{105b}$$

$$-2\ddot{\alpha} - 3\dot{\alpha}^2 - \tfrac{1}{2}\sigma_{ij}\sigma^{ij} + e^{-2\alpha}e^{-2\beta_{11}} = \frac{8\pi}{3}T^k{}_k \tag{105c}$$

$$\dot{\sigma}_{ij} + 3\dot{\alpha}\sigma_{ij} = 8\pi_{ij} \tag{105d}$$

For simplicity we take the only non zero Faraday tensor components to be $F_{23} \neq 0$ when the fluid is at rest. The fluid flow vector is chosen $U_\mu = \delta_o\delta_\mu{}^o + U_2\delta_\mu{}^2 + U_3\delta_\mu{}^3$. Then in the Lorentz force law (or by the Lorentz transformation) the magnetic field is $H' = U_oF_{23}$ and the electric field is $E_3 = F_{32}U^2$, $E_2 = F_{23}U^3$.

For the above metric the fluid kinematical quantities are:

acceleration: $A_o = \dot{U}_oU^o$,

$A_1 = 2e^{-\alpha}e^{-\beta_{11}}(U_2{}^2 + U_3{}^2)$,

$A_2 = (\dot{U}_2 + (\dot{\alpha} + \sigma_{22})U_2)U^o + (\dot{\alpha} + \sigma_{22})U_oU^2$,

$A_3 = (\dot{U}_3 + (\dot{\alpha} + \sigma_{33})U_3)U^o + (\dot{\alpha} + \sigma_{22})U_oU^3$;

expansion: $\theta = 3\dot{\alpha}$;

shear: $\Sigma_{11} = \sigma_{11}U_o$,

$\Sigma_{22} = \sigma_{22}U_o + A_2U_2$,

$\Sigma_{33} = \sigma_{33}U_o + A_3U_3$, $\Sigma_{23} = A_{(2}U_{3)}$,

$\Sigma_{20} = \tfrac{1}{2}(\dot{U}_2 + (\dot{\alpha} + \sigma_{22})U_2) + A_{(2}U_{o)}$,

$\Sigma_{30} = \tfrac{1}{2}(\dot{U}_3 + (\dot{\alpha} + \sigma_{33})U_3) + A_{(3}U_{o)}$;

rotation vector: $\omega^1 = (U_2A_3 - A_2U_3)$,

$\omega^2 = 2U_3e^{-\alpha-\beta_{11}}$,

$\omega^3 = 2U_2e^{-\alpha-\beta_{11}}$,

Reference triad rotation: $\Omega_{02} = U_{[2}A_{o]}$,

$\Omega_{03} = U_{[3}A_{o]}$,

$\Omega_{23} = U_{[2}A_{3]}$.

For MHD to exist the conductivity must be sufficiently large. Standard kinetic theory techniques give it as a function of temperature T, electron charge e and mass m by the formula[56]

$$\eta = \left(\frac{T}{300}\right)^{\frac{3}{2}}\left(\frac{e^2}{6.1 \times 10^{-9}m}\right) \tag{106}$$

For a hot intergalactic gas, perhaps as a source of the present-day x-ray background radiation,[58] the present temperature of a tenuous intergalactic medium could range 10^{10}-10^{4}degK hence the conductivity would range 10^{10}-$10^{3}\frac{\text{mho}}{\text{m}}$. This is large enough to create MHD conditions over a long period of the universe's history.

For quasi-steady-state current we have[56]

$$J_i = \upsilon_{EI}^{-1}\varepsilon_{ijk}J^J\Omega^k = -\eta F_{i\mu}U^\mu \tag{107}$$

where υ_{EI} is the electron-ion collision frequency and $\Omega^k = eB^k/M$ is the plasma cyclotron frequency. The second term in equation (107) is the Hall current term. If $\Omega^k << \upsilon_{EI}$, the Hall current will be small. It will be important here in second-order effects.

For a steady state current density $J_i = \eta E_i$ Maxwell's equations read

$$\dot{F}^{20} + (\dot{\alpha} + \sigma_{22})F^{20} = -\eta F^{20} \tag{108a}$$

$$\dot{F}^{30} + (\dot{\alpha} + \sigma_{33})F^{30} = -\eta F^{30} \tag{108b}$$

$$\dot{F}_{23} + (2\dot{\alpha} - \beta_{11})F_{23} = 0 \tag{108c}$$

with the solutions

$$F^{20} = F_0^{\ 20}e^{-\alpha-\beta_{22}-\int\eta dt} \tag{109a}$$

$$F^{30} = F_0^{\ 30}e^{-\alpha-\beta_{33}-\int\eta dt} \tag{109b}$$

$$F_{23} = F^o_{\ 23}e^{-2\alpha + \beta_{11}} \tag{109c}$$

Where the zero super and subscribts are constants and we recall the Lorentz transformation $F^{20} = F_{23}U^3 + F_{32}U^2$. In an inertial frame $E = U \times B$.

The energy conservation equation (equation 99a) becomes

$$\dot{\mu} + 3(\mu + p)\dot{\alpha} = \lambda\Sigma_{ij}\Sigma_{ij} + \eta(U_2^{\ 2} + U_3^{\ 2})B_1^{\ 2} \tag{110a}$$

The first-order momentum density equations (equation 99b) are

$$(\mu + p)A_2 = -U_2U^o\dot{p} + \eta U_2H_1^{\ 2} \tag{110b}$$

$$(\mu + p)A_3 = -U_3U^o\dot{p} - \eta U_3H_1^{\ 2} \tag{110c}$$

We take the universe to be filled with nonrelativistic matter plus a tenuous hot conducting intergalactic medium since the end of the radiation era. Then $\gamma = 1$, so $a_s = 0$. So Einstein's equations read

$$3\dot{\alpha}^2 = 8\pi\mu + \tfrac{1}{2}\sigma_{ij}\sigma^{ij} + 3e^{-2\alpha-2\beta_{11}} \tag{111a}$$

$$\mu U_o U_2 = -U_2 H_1^{\;2} \tag{111b}$$

$$\mu U_o U_3 = -U_3 H_1^{\;2} \tag{111c}$$

$$-2\ddot{\alpha} - 3\dot{\alpha}^2 = \frac{8\pi}{3}(E^2 + H^2) + \tfrac{1}{2}\sigma_{ij}\sigma^{ij} - 2^{-2\alpha}e^{-2\beta_{11}} \tag{111d}$$

$$\dot{\alpha}_{11} + 3\dot{\alpha}\sigma_{11} = 8\pi\{-\mu(U_2^{\;2} + U_3^{\;2})/3 - \lambda\Sigma_{11} - \tfrac{2}{3}H_1^{\;2} + (E_2^{\;2} + E_3^{\;2})/2\} \tag{111e}$$

$$\dot{\alpha}_{22} = 8\pi\{\mu(2U_3^{\;2} - U_2^{\;2})/3 - \lambda\Sigma_{22} + H_1^{\;2}/3 + (E_3^{\;2} - 2E_2^{\;2})/3\} \tag{111f}$$

$$\mu U_2 U_3 = \lambda\Sigma_{23} + E_2 E_3 \tag{111g}$$

We take the Friedman model to be correct to the zeroth order. We have $e^{\alpha} = (8\pi M/3)\ (\sinh^2 \frac{\tau}{\tau})$ and $\beta = 0$ where $\frac{dt}{d\tau} = e^{\alpha}$, $M = \mu_R\ (3\dot{\alpha}_R - 8\pi\mu_R)/3$ and μ_R and $\dot{\alpha}_R$ are the present values of the density and expansion respectively as in the unperturbed Robertson-Walker model.[14] We found in chapter II that $\mu_R \leq 0.1(\dot{\alpha}_R)^2$. Therefore the zero-order solution to equation (111a) is in this limit $e^{\alpha} = t$. This means that the solution to equations (111e) and (111f) is $\sigma_{ij} = A_{ij}e^{-3\alpha}$ where A_{ij} is a constant matrix. Therefore to this order

$$B_{ij} = \tfrac{1}{2}A_{ij}(t_R^{-2} - t^{-2}) \tag{112}$$

We take the magnetic fields B_i as small perturbations to the simplest possible anisotropic background. Maxwell's equation gives $B_1 = B_1^{\;o}e^{-2\alpha}$ by equation (109c). We have as our first order acceleration equation

$$\dot{U}_2 + (\dot{\alpha} + \beta_{22})U_2 = \eta U_2 H_1^{\;2}/\mu \tag{113a}$$

$$\dot{U}_3 + (\dot{\alpha} + \beta_{33})U_3 = \eta U_3 H_1^{\;2}/\mu \tag{113b}$$

These have the solution of the form

$$U_2 = U_2^{\;o}\ e^{(-\alpha-\beta_{22} + \int(\eta H_1^{\;2}/\mu)dt)} \tag{114a}$$

$$U_3 = U_3^{\;o}\ e^{(-\alpha-\beta_{33} - \int(\eta H_1^{\;2}/\mu)dt)} \tag{114b}$$

For $\mu \sim \mu^o e^{-3\alpha}$ and $H_1^{\;2} \sim H_1^{\;o2}e^{-4\alpha}$ with η a constant these give

$$U_2 = U_2{}^o(t/t_o)^{-1}(t/t_o)^{(\eta H_1{}^{o2}/\mu^o)} \qquad (115a)$$

$$U_3 = U_3{}^o(t/t_o)^{-1}(t/t_o)^{(-\eta H_1{}^{o2}/\mu^o)} \qquad (115b)$$

We now solve the inertia density equation to see the effect of Joule heating. Subject to the above solution this equation becomes (by equation (110a) with the first order approximation $\Sigma_{ij} = \sigma_{ij}$)

$$(\mu e^{3\alpha})^{\cdot} = e^{3\alpha}\{\lambda A^2 e^{-6\alpha} + \eta U_2{}^2 H_1{}^{o2} e^{-4\alpha}\}. \qquad (116)$$

This has the solution

$$\mu = \mu^o t^{-3} + t^{-3}\int\{\lambda A^2 t^{-6} + \frac{\eta H_1{}^{o2} U_2{}^{o2}}{t^4}\, t^{(2\eta H_1{}^{o2}/\mu^o)}\}t^3 dt \qquad (117)$$

This integral gives

$$\mu = \mu_o t^{-3} - t^{-3}\frac{\lambda A^2}{2}(t^{-2} - t_o{}^{-2}) + \frac{U_o}{2}\, t(2\eta H_1{}^{o2} U_2{}^{o2}/\mu^o)-3 \qquad (118)$$

In the above discussion the viscosity could have simply included by replacing $\sigma_{ij}e^{-2\alpha}$ with $\sigma_{ij}e^{-3\alpha - 8\pi\lambda t}$. We can now analyze the second order effects beginning with the Reynolds stresses and Maxwell stresses in the shear equations. We have

$$\sigma_{ij}^{II} = -e^{-3\alpha}\int 8\pi(\Pi_{ij}^R + \Pi_{ij}^M)e^{3\alpha}dt. \qquad (119)$$

This gives from the above solutions that the relevant second-order Reynolds stresses are

$$\Pi_{11}^R = -\frac{1}{3}\mu_o t^{-3}(U_3{}^{o2}(t/t_o)^{-2(1 + r)} + U_2{}^{o2}(t/t_o)^{-2(1 - r)}), \qquad (120a)$$

$$\Pi_{11}^M =$$

$$\frac{H_1{}^{o2}}{-t^4}\{(U_3{}^{o2}(t/t_o)^{-2(1 + r)} + U_2{}^{o2}(t/t_o)^{-2(1 - r)})/3 - \frac{2}{3}\}, \qquad (120b)$$

$$\Pi_{22}^R = \frac{1}{3}\mu_o t^{-3}(-U_3{}^{o2}(t/t_o)^{-2(1 + r)} + 2U_2{}^{o2}(t/t_o)^{-2(1 - r)}, \qquad (120c)$$

and

$$\Pi_{22}^M = \frac{H_1{}^{o2}}{t^4}\{(U_2{}^{o2}(t/t_o)^{-2(1 - r)} - 2U_3{}^{o2}(t/t_o)^{-2(1 + r)})/3 + \frac{1}{3}, \qquad (120d)$$

where $r = \eta H_1{}^{o2}/\mu^o$. The contribution from each of the above quanities to σ_{ij}^{II} is

$$\sigma_{11}^{IIR} = -\frac{1}{t^3}\int 8\pi\{-\frac{1}{3}\mu_o t^{-3}(U_3{}^{o2}(t/t_o)^{-2(1 + r)} + U_2{}^{o2}(t/t_o)^{-2(1 - r)})t^3\}dt, \qquad (121a)$$

$$\sigma_{11}^{IIM} = -\frac{1}{t^3}\int 8\pi\{\frac{H_1^{o2}}{t^4}(\{U_3^{o2}(t/t_o)^{-2(1+r)}$$

$$+ U_2^{o2}(t/t_o)^{-2(1-r)}\}/3 - \frac{2}{3})t^3\}dt, \quad (121b)$$

$$\sigma_{22}^{IIR} = -\frac{1}{t^3}\int 8\pi\{\frac{1}{3}\mu_o t^{-3}(-U_3^{o2}(t/t_o)^{-2(1+r)} + 2U_2^{o2}(t/t_o)^{-2(1-r)})t^3\}dt, \quad (121c)$$

and

$$\sigma_{22}^{IIM} = -\frac{1}{t^3}\int 8\pi\{\frac{H_o^{o2}}{t^4}(\{U_2^{o2}(t/t_o)^{-2(1-r)}$$

$$- 2U_3^{o2}(t/t_o)^{-2(1+r)}\}/3 + \frac{1}{3})t^3\}dt, \quad (121d)$$

where $r = \eta H_1^{o2}/\mu^o$. The above intergrals have solutions as follows

$$\sigma_{11}^{IIR} = \frac{8\pi\mu_o}{3t^3}\{\frac{U_3^{o2}t^{-2(1+r)+1}}{t_o^{-2(1+r)}(-1-2r)} + \frac{U_2^{o2}t^{-2(1-r)+1}}{t_o^{-2(1-r)}(-1+2r)}\}, \quad (122a)$$

$$\sigma_{11}^{IIM} = \frac{-8\pi H_1^{o2}}{3t^3}\{\frac{U_3^{o2}t^{-2(1+r)}}{t_o^{-2(1+r)}(-2-2r)} + \frac{2U_2^{o2}t^{-2(1-r)}}{t_o^{-2(1-r)}(-2+2r)} - \frac{2}{3}\ln t\}, \quad (122b)$$

$$\sigma_{22}^{IIR} = -\frac{8\pi\mu_o}{3t^3}\{-\frac{U_3^{o2}t^{-2(1+r)+1}}{t_o^{-2(1+r)}(-1-2r)} + \frac{2U_2^{o2}t^{-2(1-r)+1}}{t_o^{-2(1-r)}(-1+2r)}\}, \quad (122c)$$

and

$$\sigma_{22}^{IIM} = -\frac{8\pi H_1^{o2}}{3t^3}\{\frac{U_3^{o2}t^{-(1+r)}}{t^{-2(1+r)}(-2-2r)} - \frac{2U_2^{o2}t^{-2(1+r)}}{t^{-2(1-r)}(-2+2r)} + \frac{1}{3}\ln t\}, \quad (122d)$$

where $r = \eta H_1^{o2}/\mu^o$. The second-order momentum density equation is

$$(\mu + p)A_1 = -(J \times B)_1 - \Pi_1^k{}_{||k} \quad (123)$$

The second-order current J_i is the Hall current J_i^H; this is $J_i^H = -\upsilon_{E_I}^{-1}\varepsilon_{ijk}J^J\Omega^k$, which gives

$$J_2^H = -\upsilon_{EI}J^3\Omega^1 \quad (124a)$$

and

$$J_3^H = -\upsilon_{EI}^{-1}J^2\Omega^1 \quad (124b)$$

Therefore the Lorentz force term in equation (123) vanishes.

The stress divergence term $-\Pi_1^k{}_{||k}$ is

$$2\lambda\Sigma_1^1 e^{-\alpha-\beta_{11}} + \lambda\Sigma_1^o - \lambda\Sigma_{11}^o\Gamma_{10}^1 \quad (125)$$

Inserting this in equation (123) above we find an equation of evolution for A_1:

$$\dot{A}_1 + A_1(\dot{\alpha} + \sigma_{11}) - \frac{\mu}{\lambda}A_1 = 2\Sigma_{11}e^{-\alpha-\beta_{11}} \quad (126)$$

This has the solution

$$A_1 = A_1{}^o t^{-1} e^{-\beta_{11}} e^{\mu/2\lambda t^2} - \frac{\lambda A_{11}}{2\mu^o t} e^{-\beta_{11}} e^{4\mu^o/\lambda t^2} \tag{127}$$

This equation is consistent with the definition of A_1 in the definitions of the kinematic quantities. Further the $G_{oi} = -8\pi T_{oi}$ and $G_{23} = -8\pi\Pi_{23}$ give consistency conditions. (These are equations (111b) and (111c) G_{oi}, and (111g) G_{23}). These are compatible to all orders of approximation used in this thesis.

We finally calculate the effects of these perturbations on the isotropic expansion. We expand equation (111a) via $\alpha \to \alpha + \delta\alpha$ with $(\delta\alpha)^2 << 1$. Then we have

$$(\delta\alpha)^{\cdot} = (2\dot{\alpha})^{-1} \{\frac{1}{2} \sigma_{ij}\sigma^{ij} - 6\beta_{11}e^{-2\alpha}\} \tag{128}$$

which is, with the above results for the quantities in brackets

$$(\delta\alpha)^{\cdot} = \frac{t}{2} \{\frac{A^2}{2}t^{-6} - \frac{A_{11}}{t^2} (\frac{1}{t_o{}^2} - \frac{1}{t^2}) \}. \tag{129}$$

We therefore find the solution

$$\delta\alpha = - \frac{A^2}{16t^4} + \frac{3A_{11}}{t^2} (\frac{1}{t_o{}^2} - \frac{1}{4t^2}). \tag{130}$$

Finally we find for the rotation in these models from the kinematical quantities' definitions:

$$\omega^1 = -2\eta U_2{}^o U_3{}^o (t/t_o)^{-8} \tag{131a}$$

$$\omega^2 = 2U_3{}^o (t/t_o)^{-(2 + r)} \exp(A_{11}(\frac{1}{t_o{}^2} - \frac{1}{t^2})), \tag{131b}$$

and

$$\omega^3 = 2U_2{}^o (t/t_o)^{-2(2 - r)} \exp(A_{11}(\frac{1}{t_o{}^2} - \frac{1}{t^2})), \tag{131c}$$

where $r = \eta H_1{}^{o2}/\mu^o$.

This completed the examination of the model. We have determined all the kinematics and dynamics of fluids, fields and geometry. We can now make numerical estimates based on the observations.

Observations

It is of interest to obtain some numerical estimates of the kinematic quantities and present dynamics of the model. The most accurate global cosmological datum at present is the microwave blackbody background radiation. In our analysis of this radiation and the information it carries about the universe we follow the approach of Collins and Hawking.[14]

To first order the temperature measured for the microwave background received (R) at the present time from the emitting surface (E) of last scattering of the radiation is

$$T_R = T_E e^{\alpha_E - \alpha_R} \{1 + p^i U_i |_E^R - \int_E^R p^i p^j \sigma_{ij} dt\} \tag{132}$$

The monopole contribution is simply $e^{\alpha_E - \alpha_R}$ giving the isotropic part of T_R. The dipole variation δT_D is $p^i{}_R U_{Ri} - p^i_E U_{Ei}$ where the first part refers to the present tangent geodesic vector (direction cosine) p^i_R and the present flow velocity U_R. The second part is for those quantities at the emitting surface. The quadupole variation δT_q is the integral involving the shearing of the flow from (E0 to (R).

For the geodesics in the isotropic background model we have with $K_1 = K\cos\theta$, $K_2 = K(\sin\theta)(\cos\phi)$ and $K_3 = K(\sin\theta)(\cos\phi)$ that in the E^A_μ frame

$$\phi = \phi_o \tag{133a}$$

$$K = K_o e^{-\alpha} \tag{133b}$$

$$\theta = 2\cot^{-1} B(t-t_o) + C \tag{133c}$$

where B is a constant and $C = \cot\theta_o$. The term $P^i{}_R U_{Ri}$ is then of the form

$$T_E e^{-\alpha_E - \alpha_R}\{U_{R2}(\sin\theta)(\cos\phi) + U_{R3}(\sin\theta)(\cos\phi)\} \tag{134a}$$

while for $P^i{}_E U_{Ei}$ we have

$$-T_E\{1 + \cot^2(\tfrac{\theta}{2}) e^{\alpha_R - \alpha_E}\}^{-1}\{2\cot(\tfrac{\theta}{2}) e^{\alpha_E - \alpha_R}(U_{R2}\cos\phi + U_{R3}\sin\phi)\} \tag{134b}$$

The quadrupole term gives after integration with $\cot\frac{\theta}{2} = t$ and $\sigma_{ij} = A_{ij} t^{-3}$:

$$T_E e^{-\alpha_E - \alpha_R}\{A_{11}\{\frac{\cos\theta}{4} + \frac{2}{\cos^3\frac{\theta}{2}}(\sin^2\frac{\theta}{2} - \frac{2}{3}) + \frac{\sin^4\frac{\theta}{2}}{4\cos^2\frac{\theta}{2}} - \tan^2\frac{\theta}{2}$$

$$-2\ln\cos\frac{\theta}{2}\} - \frac{4}{\cos^2\frac{\theta}{2}}(A_{22}\cos^2\phi + A_{33}\sin^2\phi)(\sin^2\frac{\theta}{2} - \frac{2}{3})\}\Big|_E^R \tag{134c}$$

The most accurate microwave background temperature anisotropy measurement to date is that of Smoot <u>et al</u>.[59] although accurate earlier measurements have been made scanning different circles in the sky.[60-64] Smoot <u>et al</u>.[59] find

$$\delta T_D = 1.296 \times 10^{-3} T_R \tag{135a}$$

and

$$\delta T_Q \leq 3.703 \times 10^{-4} T_R \tag{135b}$$

These values give for the present velocity $U_R = (U_{Ri}U_R{}^i)^{\frac{1}{2}}$ of

$$U_R \simeq 5 \times 10^{-4} \tag{136a}$$

and for the R.M.S. shear $\sigma = (\frac{1}{2}\sigma_{ij}\sigma^{ij})^{\frac{1}{2}}$

$$\frac{\sigma}{\dot\alpha} \leq 3.7 \times 10^{-5} \tag{136b}$$

Then the rotation is, from its dependance on U_2 and U_3 is, for ω_2 and ω_3, with $\dot\alpha = e^{-\alpha}$

$$\omega_2, \omega_3 \lesssim 10^{-14} \text{rad/yr.} \tag{137}$$

The shear limits give

$$\sigma \lesssim 3.7 \times 10^{-15} \text{yr}^{-1} \tag{138}$$

so we may use both equations (136) and equations (122a) (122d) to set limits on $H_1{}^R$ and also check for consistency of the U_R measurement with the global shear limits. We find

$$H_1{}^R \lesssim 10^{-8} \text{gauss.} \tag{139}$$

Next the Reynold's stresses contribute to σ_R as approximately

$$\sigma_R{}^R \sim 3^{-3\alpha} \int \mu_R U_R{}^2 e^{3\alpha} d(e^\alpha) \tag{140}$$

which gives

$$\sigma_R{}^R \sim \mu_R U_R{}^2 e^{-2\alpha_R} \tag{141}$$

This means that μ_R must satisfy

$$\mu_R \lesssim 1.6 \times 10^{-31} gm/cm^3 \tag{142}$$

which is what we required in solving the equations initially. Thus, the solutions are consistent with each other and our numerical estimates from the data. In fact equation (142) says that $\Omega \lesssim 0.1$.

<u>Discussion</u>

We have examined a Bianchi V anisotropic spatially homogeneous cosmological model. Although the metric is diagonal we have introduced a small magnetic field against the isotropic open Friedman model. The field direction was chosen orthogonal to the plane in which the invariant vector fields ${E_2}^\mu$ and ${E_3}^\mu$ lie. The fluid flow vector was taken to lie in that plane. It and the shear tensor are introduced as first-order perturbations. To second-order the Joule heating and effects of Reynolds and Maxwell stresses on the normal shear were studied. Rotation appeared as a first-order effect and posessed second-order components. A very important effect was the Lorentz force contribution to the fluid flow, which strongly accelerated the component U_2 and strongly decelerated the component U_3.

The acceleration and deceleration of U_2 and U_3 assure that the models <u>do</u> <u>not</u> evolve into a locally rotationally symetric configuration and that the second-order rotation component ω^1 is not zero although it decreases rapidly (in fact as t^{-8}).

The model is quite reasonable in most aspects. We adopt a low density open Friedman model as a background consistent with density parameter $\Omega = 0.1$. In such a model the isotropic expansion goes as $e^\alpha = t$. We found the shear evolving as $\sigma_{ij} \sim A^o_{ij}/t^3$ plus second order Reynolds stresses and Maxwell stresses.

Numerical estimates from the limits on quadrupole and dipole anisotropy of the microwave background radiation fixed present values of flow U_2^R, U_3^R - 5×10^{-4}: rotation ω_2^R, $\omega_3^R \leq 10^{-4}$rad/yr, $\omega_1^R \leq 10^{-18}$rad/yr; and $|\sigma_{ij}| \leq 3.7 \times 10^{-15}\text{yr}^{-1}$ with a Hubble parameter $\dot{\alpha}_R \sim 10^{-10}\text{yr}^{-1}$; the magnetic field $H_i^R \leq 10^{-8}$gauss; and therefore the present value of the matter density $\mu p \leq 1.6 \times 10^{-31}\text{gm cm}^{-3}$ in good agreement with the observed condition $\Omega \leq 0.1$.

So we see that an MHD open universe is a quite reasonable model of the universe. This model is important as the observations presently seem to indicate that the universe is open and plasma processes must have been important when the universe was radiation-dominated. In addition MHD processes would be important if a magnetic field were present, even in the baryon-dominated era if there were a tenuous hot intergalactic gas.

This is important as galaxies cannot form in an open universe by purely gravitational interactions.[65] In this spirit the near-isotropy of the universe and the fact that galaxies exist at all seems a contradiction if the universe is open.[66] It is appealing then to examine whether MHD processes can affect galaxy formation in an open universe. Such is possible as the MHD processes induce a local Bianchi-type breaking curvature change in the fluid dynamics.

For example, consider the fluid flow first. We find

$$U_2 = U_2{}^o(t/t_o)^{-(1-\eta H_1{}^{o2}/\mu^o)} \tag{143}$$

indicating a body force acceleration. Then in the inertia density conservation equation the Joule heating contributed the density perturbation:

$$\delta\mu = \mu_o/2\ t^{2(\eta H_1{}^{o2}/\mu^o)U_2{}^{o2}-3} \tag{144}$$

we can thus build up a large turbulent flow and inertia concentrations via equations (143)-(144), so long as the conductivity is sufficiently large. If the temperature is just 10^4degK it will be 10^3Mho/M, large enough to keep growing increasingly. Thus MHD open universe cosmologies offer an attractive resolution of the dilemma of how to make galaxies in a universe that has always been expanding too rapidily to let them condense out of it.

CHAPTER V. CONCLUSIONS

The results given in the study are very encouraging. The existence of galaxies and the universe being open seem mutually exclusive.[65,66] Yet galaxies exist and the evidence is strong that the universe is open. MHD processes might provide the disturbing mechanism (a "pseudocurvature") to allow clumps to grow to make galaxy "seeds" against an open background. We have seen that as long as the conductivity η is large we have strong polynomial growth of condensations $\delta\mu$,

$$\frac{\delta\mu}{\mu} \sim (t/t_o)^{2(\eta {H_1^o}^2/\mu^o)}, \tag{145}$$

relative to the background density μ. Also the MHD acceleration will drive the velocity disturbance δU_2 beyond that for a background with magnetic field only U_2 by

$$\frac{\delta U_2}{U_2} \sim (t/t_o)^{\eta {H_1^o}^2/\mu^o} \tag{146}$$

thus providing a source of turbulence to give density condensations by viscous decay.

A theorem of Hughston and Jacobs[67] might spoil this scheme by showing that magnetic fields are not admissible in Bianchi V cosmologies. For source-free, diagonal Bianchi V cosmologies with $U_\mu = U_o\delta^o_q$, their theorem is true. But the admission of <u>both</u> source terms in Maxwell's equations (J_μ) and a nonzero peculiar velocity υ_i allows the magnetic field, as discussed in Appendix I.

Finally we have found that MHD processes in these models are consistent with approaching isotropy in the models as the shear can presently be quite small as can be any present magnetic field, flow and rotation. Yet all of these quantities could have been quite large in the past, approaching $\sigma_E/\dot{\alpha}_E \sim \omega_1{}^E/\dot{\alpha}_E \sim U_E \sim 1$ and $H \sim 10^{10}$ gauss at large redshifts deep into the radiation era since these quantities go as $(t/t_o)^{-2}$ and thus grow rapidly as we return to the past of the universe.

Lastly we note that the conductivity η may still be high if a tenuous hot intergalactic medium exists. The actual form of the conductivity is $\eta = Ne^2/m\upsilon_{E_I}$ where N is the particle number density and $\upsilon_E = 6.1 \times 10^{-4} N \times (300/T)^{\frac{3}{2}}$ is the collision frequency.[56] Clearly the temperature T is the major contributor until the magnetic field drives the cyclotion frequency to be large. But then the Hall conductivity $\eta eB/M\upsilon_{E_I}$ will be large and so anomalous Hall currents will still provide MHD processes.[56]

We conclude that more general models of Bianchi V and VII are worth the effort of their developement following these promising results.

APPENDIX I

ON THE THEOREM OF HUGHSTON AND JACOBS

Hughston and Jacobs have proven a theorem that diagonal Bianchi V cosmologies may not possess a magnetic field. However their theorem is severely restricted:

1. There are no currents (source-free Maxwell Equations, *i.e.*, $F_{\mu\upsilon}{}^{||\upsilon} = 0$).
2. There is no Poynting vector, *i.e.*, $P_i = \varepsilon_{ijk}E^jB^k = 0$.
3. There is no electric field, $E_i = 0$.

In our model, with $U_i \neq 0$, and a finite conductivity $\eta \neq 0$ we have $J_i \neq 0$, $E_i = \varepsilon_{ijk}U^jB^k \neq 0$, and $P_i = \varepsilon_{ijk}B^k \neq 0$. Then Maxwell's equations retain sufficient freedom that $H_i(=B_i/m)$ is not required to vanish, just as in the Einstein equations with a diagonal Bianchi V metric $P_i \neq 0$ indicates $U_i \neq 0$. MHD effects thus endow the model with a richer dynamics and imitate the presence of a richer structure in the geometry.

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