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A Monte Carlo Analysis of Thorndike's Indirect Range Restriction Correction Equations

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A MONTE CARLO ANALYSIS OF THORNDIKE'S INDIRECT RANGE RESTRICTION CORRECTION EQUATIONS

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I dedicate this thesis to my parents, Mike and Shawna Pelayo; and grandparents, Martin and Darlene Pelayo. From an early age, each of you instilled in me the importance of a strong education. I would not have experienced the success I have had without first learning from you all. I will always be grateful for that.

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Employee selection is an important process for organizations. Organizations seek to select the best employees for their available positions. Testing is key to many selection efforts. The results of studies assessing the criterion-related validity of a selection test are affected by a number of statistical artifacts, one of which is range restriction. Range restriction has the effect of attenuating the correlation coefficient. Statistical equations exist to correct for the effects of range restriction, and they enable researchers to obtain a more accurate estimate of the validity coefficient. Thorndike (1949) developed the best known and most frequently used of these correction equations. In the present study, Monte Carlo analyses were used to compare the accuracy of two indirect range restriction correction equations. The only difference between the two equations is the nature of the predictor intercorrelation employed; one equation uses the restricted predictor intercorrelation, whereas the other uses the restricted value. The distinction between these values is important as both forms of the correlation are likely available in a predictive design, and the magnitude of each can be quite different depending on the extent of range restriction. Given these differences between the two forms of the equation, I hypothesized that the equation utilizing an unrestricted predictor intercorrelation would be more accurate. Results indicated that the equation that made use of the unrestricted correlation was generally more accurate, particularly when the selection ratio was low, and the predictors were not highly correlated.

Introduction

Organizations can take many shapes and sizes, ranging from a small municipal government to a large, publicly traded company. The cornerstone of any organization is its employees. An organization selects employees based on the belief that those chosen will serve to benefit the organization and help with the achievement of their mission and goals (Guion, 1998). Therefore, personnel selection has been a major topic of research in the field of industrial-organizational psychology (Schmidt & Hunter, 1998). In order to find optimal employees, organizations may test applicants with a carefully designed array of exams to determine whether or not an applicant will be successful on the job. Applicants with high scores on the exams are expected to display higher levels of job performance once on the job. Industrial-organizational psychologists are typically employed by many companies to determine if the aforementioned tests are successful at predicting future job performance, or even to find new tests that predict job performance through the use of a criterion-related validity study (Schmidt, Ones, & Hunter, 1992).

A validity study first begins with a job analysis. A job analysis is the systematic study of a job to determine the responsibilities and requirements for a particular position. When conducting a job analysis, analysts collect information related to the tasks that employees perform, the KSAs needed to complete tasks, and the equipment typically used by jobholders, among other pieces of information. This information is compiled and can be used for a variety of typical human resources functions such as the alignment of selection test procedures with the job specification information that was obtained during the job analysis (Brannick, Levine, & Morgeson, 2007). Next, there is a statement of the proposed use of a particular test, the intended interpretations of test scores, and expected outcomes. A criterion-related validity study is a type of study in which evidence for validity is presented by the demonstration of a relationship between scores on a selection test (predictor) and a measure of work outcomes, known as the criterion (SIOP, 2018). This is done in order to obtain the operational validity of the selection test, which is an estimate of how well the predictor in practical use (e.g., when selecting job applicants) correlates with the criteria in question (Brown, Oswald, & Converse, 2017). In order to demonstrate this relationship, selection test scores are correlated with a criterion, such as job performance scores. The expected outcome is that the applicants with high scores on the test will have higher job performance scores. If this occurs, then the test is shown to be effective for its intended use. For the purpose of selection, these studies are performed in order to gather validity evidence that supports a certain interpretation of test scores, such as the prediction that high test scores will lead to higher observed job performance. The greater the evidence, the more likely the organization is to accept the findings and use the selection test in the future when selecting employees.

Criterion-related validity studies can be conducted using two distinct designs: predictive and concurrent. When using a predictive design, scores on a selection test are obtained first, from job applicants. Then, at a later date when applicants have transitioned to full-time employment, criterion scores (typically on some measure of job performance) are collected. Scores on the predictor and criterion are then corrected in order to obtain the predictor test's validity. In a concurrent validity study design, scores on the predictor and criterion are obtained at approximately the same time from employees in the organization. This is a distinct difference between predictive and concurrent designs. A predictive design collects predictor data from applicants and criterion data from them once they are actual employees, whereas a concurrent design collects both from the current pool of workers. These different designs have implications for the validity coefficient obtained in the study. When a predictive design is utilized, applicants will probably be motivated to perform well on the selection test in order to get the job. A concurrent design uses only current job holders, in other words, people who may not share the same level of motivation because they are already employed by the organization and do not see any potential benefit exerting all of their effort on the test. This score distortion will affect the data and the validity coefficient.

A criterion-related validity study is feasible if: there are adequate sample sizes, sufficient score ranges within the predictor and criterion, and an unbiased, relevant, and reliable criterion (Guion, 1998). An adequate sample helps to generalize study results to a larger population. Sufficient score ranges bring more variance that enable uncorrected coefficients to be closer to the actual population value. An acceptable criterion measure is needed so that the measure chosen actually provides a good approximation of the construct in question.

Validity is the most important aspect to consider when attempting to develop and determine the effectiveness of selection procedures (SIOP, 2018). If a selection procedure lacks validity, it is essentially useless and may lead to lawsuits against an organization if it continues to be used once evidence against it appears. Despite the best efforts of researchers, there are statistical artifacts that can serve to decrease the size of the estimated validity coefficient obtained from a validity study. One such artifact that can decrease a validity coefficient is range restriction (Schmidt, Hunter, & Urry, 1976). Other important artifacts to consider when conducting a validation study are predictor and criterion reliability and sampling error, as large sample sizes are needed to ensure adequate statistical power. Industrial-organizational psychologists strive to obtain accurate estimates of validity coefficients in order to accurately determine a selection test's relation with job performance. Therefore, it is necessary and important to employ correction procedures, through the use of statistical equations, to account for the effects of range restriction on the obtained validity coefficients.

Range Restriction

Range restriction occurs when a researcher is seeking to estimate the relationship between two variables (e.g., selection test and job performance), but score variability on one or both variables is reduced because of selection decisions. This reduction in variability affects the validity coefficient by distorting its size (Raju & Brand, 2003). When the variance on either the predictor or criterion is smaller in the selected sample than in the relevant population, the sample coefficient will underestimate population validity (Guion, 1998). In essence, researchers desire to find a relationship between X (the predictor) and Y (the criterion to be predicted) in an unrestricted population, but the only data available are from a restricted sample (Mendoza & Mumford, 1987).

Although selection can result in correlations of greater strength than in the population (e.g., if the middle of the distribution of scores are removed), the nature of selection in applied personnel selection (i.e., top-down selection) serves to reduce the strength of correlations in the selected sample. In addition to the nature of selection, range restriction may increase the correlation between two variables or even show no effect on the relationship at all if the relationship between the predictor and criterion variable is not linear (Huck, 1992).

To illustrate a situation where range restriction occurs, consider the following scenario. An organization is attempting to validate a test they believe can estimate future job performance. Those who score higher on the test are expected to perform the job better and selection decisions are made using this test. The sample of data available for the validity study only includes applicants who were actually hired. Thus, low scoring applicants are not included in the analysis, reducing the variability of scores on both predictor and criterion. This reduced variability (i.e., range restriction) reduces the resultant correlation between test scores and job performance (Thorndike, 1949). The range of scores on the selection test is restricted; therefore, the correlation between test scores and job performance can only be obtained from the restricted sample. In a situation like this, the obtained correlation is expected to be an underestimate of the population correlation (Henriksson & Wolming, 1998).

Types of Range Restriction

Range restriction in selection can take two forms, direct (also called explicit) or indirect (also called incidental). Direct range restriction occurs when applicants are selected top-down by test scores (Hunter, Schmidt, & Le, 2006; Sackett & Yang, 2000; Wiberg & Sundström, 2009). When an organization selects applicants top-down on a given selection test, there are no low scoring applicants hired; therefore, those applicants are not included in a criterion-related validity study as they do not have criterion scores. Thus, a validity coefficient can only be computed from applicants who were hired. Consider college applications for a hypothetical United States university called Big State University. BSU is an institution known for its extremely rigorous admission standards. High School students must have high GPAs and standardized test scores to be admitted. Big State University will not accept students who fail to meet this high threshold. In correlating high school GPA or test scores with first year college GPA, a weak correlation would be found, contrary to expectations. As BSU only selects students with high test scores and GPAs, there is little variance in the data. This low level of variance as compared to the applicant sample is a restriction of range that leads to a low correlation.

Indirect range restriction is the second form of range restriction. In this scenario, applicants are chosen on the basis of some other third variable (*Z*) that is correlated with the predictor variable (*X*) to some extent (Hunter et al., 2006; Wiberg & Sundström, 2009; Zimmermann, Klusmann, & Hampe, 2017). As an example, to illustrate indirect range restriction in action, consider the Graduate Record Examination (GRE). The GRE is a test designed by the Educational Testing Service (ETS) and is intended to predict future performance of graduate school students (Kuncel, Hezlett, & Ones, 2001). Numerous studies have found the GRE to be a valid predictor of graduate school performance (Broadus & Elmore, 1983; Kuncel et al., 2001; Sleeper, 1961); therefore, it is typically used for graduate school selection decisions. Top-down selection of school applicants by GRE scores would result in direct range restriction. If graduate school applicants were instead selected on the basis of a variable correlated with GRE scores (e.g., undergraduate GPA), then the nature of the range restriction would be indirect (Hunter et al., 2006).

Correcting for Range Restriction

The Standards for Educational and Psychological Testing (AERA, APA, & NCME, 1999) and the Principles for the Validation and Use of Personnel Selection Procedures (SIOP, 2018) recommend the adjustment of validity coefficients. The Principles state, "when range restriction distorts validity coefficients, a suitable bivariate or multivariate adjustment should be made when the necessary information is available" (SIOP, 2018, p. 14). If the assumptions regarding correction formulas are met, the adjusted coefficient is the best estimate of the population validity coefficient (SIOP, 2018).

Researchers have been examining range restriction and methods for correcting validity coefficients since Pearson's (1903, 1908) work on correlations. Aitken (1934) and Lawley (1943) supplemented these early works by developing formulas that could be applied to multivariate cases of range restriction. Thorndike (1949) improved developed correction methods for direct and indirect range restriction. Corrections for range restriction have been conducted in a variety of scenarios such as test validation, selection, and, in more recent memory, validity generalization studies (Hedges & Olkin, 1985).

Thorndike's (1949) work on range restriction correction formulas has been studied over the years by a variety of researchers (Duan & Dunlap, 1997; Holmes, 1990; Linn, 1983; Ree, Carretta, Earles, & Albert, 1994). Although others have attempted to develop other correction methods, Thorndike's three correction equations (referred to as Cases 1, 2, and 3 in the original literature) are still consistently used in the field (Hunter et al., 2006). It is important for researchers to use the correct formula when correcting for range restriction, as correction methods employed under the wrong conditions may alter the corrected validity coefficient in potentially damaging ways (Alexander, Carson, Alliger, & Barrett, 1984).

Correcting for Direct Range Restriction

Thorndike's (1949) Case 2 formula for correcting for direct range restriction is presented below.

$$R_{xy} = \frac{r_{xy}(\frac{S_X}{s_x})}{\sqrt{1 + r_{xy^2}(\frac{S_X^2}{s_x^2} - 1)}}$$

As mentioned, direct range restriction involves only two variables: *X*, the predictor variable through which selection was executed, and *Y*, the criterion. R_{xy} is the corrected (thus, unrestricted) coefficient, r_{xy} is the correlation from the restricted sample, S_X and s_x represent predictor standard deviations for unrestricted and restricted groups. This equation corrects for univariate range restriction, a scenario involving truncation of just a single variable, the predictor *X*. Direct range restriction can occur by selecting applicants above a certain cutoff score and rejecting those applicants who score below it.

Correcting for Indirect Range Restriction

Thorndike (1949) presented two equations to correct for indirect range restriction; these equations are the focus of the present study. These equations are also commonly referred to as Thorndike's Case 3.

$$R_{xy} = \frac{r_{xy} + r_{xz}r_{yz}\left(\frac{{S_Z}^2}{{s_z}^2} - 1\right)}{\sqrt{\left[1 + {r_{xz}}^2\left(\frac{{S_Z}^2}{{s_z}^2} - 1\right)\right]\left[1 + {r_{yz}}^2\left(\frac{{S_Z}^2}{{s_z}^2} - 1\right)\right]}}$$

In this equation: r_{xy} is the restricted correlation between the experimental predictor and the criterion, r_{yz} is the restricted correlation between the criterion and the operational predictor, r_{xz} is the restricted correlation between the experimental and the operational predictors, ${S_Z}^2$ is the unrestricted variance of the operational predictor, and ${s_z}^2$ is the restricted variance of the operational predictor. This equation demonstrates subjects being selected on the operational predictor *Z*, a third variable with a relationship to the experimental predictor *X*.

The second indirect range restriction correction equation proposed by Thorndike (1949) uses the unrestricted predictor intercorrelation (i.e., the correlation between *X* and *Z*). This second equation is listed below.

$$R_{xy} = \frac{r_{xy}\sqrt{\left[1 + R_{xz}{}^{2}\left(\frac{s_z{}^{2}}{S_Z{}^{2}} - 1\right)\right]} + R_{xz}r_{yz}\left(\frac{S_Z}{s_z} - \frac{s_z}{S_Z}\right)}{\sqrt{\left[1 + r_{yz}{}^{2}\left(\frac{S_Z{}^{2}}{s_z{}^{2}} - 1\right)\right]}}$$

Aside from the substitution of the restricted predictor intercorrelation (r_{xz}) with the unrestricted value (R_{xz}), all terms in this equation are the same as the first equation. In a job selection scenario, R_{xz} will be available in unrestricted form if all applicants take both selection tests.

Issues with Range Restriction

Corrections for range restriction are based on three assumptions: the linearity of regression of *Y* on *X*, homoscedasticity of error distributions, and normally distributed variables (Greener & Osburn, 1979). Lawley (1943, as cited in Greener & Osburn, 1979), however, reported that the assumption of normality is not necessary.

Lee, Miller, and Graham (1982) corrected validity coefficients for the relationship between the Navy Basic Test Battery and the Navy Enlistment Exam under five different selection ratios, finding the corrected coefficients to be slightly overcorrected but, still providing better figures than the uncorrected coefficients. Brown, Stout, Dalessio, and Crosby (1988) also found evidence of overcorrections, citing violations of assumptions as the potential reason and urging that all aspects of the predictor-criterion relationship be examined. Overcorrection is more of a concern to researchers than under-correction, as it will lead to an overstatement in the predictive value of a test (Linn, Harnisch, & Dunbar, 1981).

Greener and Osburn (1979) studied the accuracy of corrections for direct range restriction in distributions that violated the assumption of linearity, homoscedasticity, or both. Through their corrections, they found that for small correlations between .10 and .25, correlations corrected were generally no more accurate than the uncorrected counterparts. For correlations ranging from .30 to .60, corrected coefficients were more accurate than uncorrected ones. Corrections for direct range restriction seem to be sensitive to violations of linearity but insensitive to homoscedasticity violations. Similar findings were reported by Gross and Fleischman (1983). Lord and Novick (1968) noted that violations of assumptions are inclined to happen frequently in professional practice and that these violations will mostly have only minor effects on the obtained coefficients. Nevertheless, researchers still encourage the use of range restriction correction equations due to the fact that uncorrected coefficients are more biased than those that are corrected (Gross & Kagen, 1983).

The Monte Carlo Method

The Monte Carlo method (also known as Monte Carlo simulations or analyses) is a statistical technique ideal for investigating the range restriction correction equations and their effectiveness. The Monte Carlo Method was developed by Metropolis and Uslam (1949) in the 1940s while they worked at Los Alamos Laboratory in New Mexico, a site organized for the eventual creation of the Atomic Bomb. The Monte Carlo method allows for the creation of large datasets that can be manipulated. In the case of range restriction correction equations, many variables can be manipulated for use in the equations, such as the selection ratio, predictor intercorrelation, the correlation between the criterion and operational predictor, and the correlation between the experimental predictor and the criterion. Large population sizes (e.g., a million distinct cases) can be generated through the Monte Carlo method. Regarding range restriction, a complete population dataset can be generated with a known population correlation between the predictor and criterion. Samples can then be drawn from this population, allowing a sample correlation to be computed. Next, the aforementioned variables would be manipulated and inserted into the equations. The equations would produce a corrected validity coefficient and the obtained value would be subject to comparison with the true population correlation in order to see the effectiveness under the various conditions present (e.g., different selections ratios and predictor intercorrelation).

The Monte Carlo method is extremely useful as it does not deal with real subjects, freeing the researcher from limitations imposed by practical considerations (e.g., difficulty in collecting sufficiently large samples). Monte Carlo analyses rely solely on computer algorithms for data creation. They can explore a variety of possible conditions related to the correction equations such as the level of range restriction and the strength of correlations. A disadvantage of the Monte Carlo method is that the conditions explored during the course of the analysis may not mirror actual conditions in a real-world setting.

Current Study

Thorndike (1949) offered two different methods for use in correcting indirect range restriction. The equations are similar with the only differences due to the nature of the predictor intercorrelation; the correlation between X and Z is restricted in one version, whereas it is unrestricted in the other. When correcting for indirect range restriction, the equation using the restricted predictor intercorrelation is commonly used (Zimmerman et al., 2017) and is often the only version of the correction printed in personnel psychology textbooks (cf. Guion, 1998). Thorndike's unrestricted predictor intercorrelation equation has the potential to provide more accurate results (i.e., estimates of the unrestricted correlation between X and Y that are closer to the actual unrestricted value) than the more common form that uses the restricted predictor correlation. There are two reasons to hypothesize this greater accuracy. First, a larger sample size is employed in the unrestricted predictor intercorrelation, thereby reducing the effects of sampling error. Second, use of the unrestricted correlation avoids the additional error that arises from any range restriction correction; in the case of the common Thorndike correction, the restricted correlation between X and Z is itself corrected for direct range restriction so that the correlation between X and Y may be corrected for indirect range restriction.

When selection decisions are made in organizations, the unrestricted predictor intercorrelation is available if all applicants take both tests (i.e., the operational and experimental predictors). The present study employed Monte Carlo techniques to investigate the accuracy of the two indirect range restriction equations developed by Thorndike (1949). It is hypothesized that the indirect range restriction equation that uses the unrestricted predictor intercorrelation will be more accurate than the form of the equation utilizing a restricted predictor intercorrelation.

Four variables were manipulated in this study: the selection ratio and the three bivariate correlations between X, Y, and Z. Correlations from a range restricted sample were computed and adjusted with both indirect range restriction correction equations.

Accuracy of the equations were determined by comparing the corrected sample value with the known population value.

Method

Conditions

Four variables were manipulated in this study. The selection ratio, the proportion of the number of hired to the job to the number in the applicant sample, was set to .01, .1 and .5 (e.g., when the selection ratio = .5, 300 cases were randomly selected from the population to serve as the applicant sample, 150 of which were selected for the hired sample). The population correlation between the experimental predictor and the criterion (i.e., R_{XY}) was set to .3 and .5. The population correlation between the operational predictor and experimental predictor (i.e., R_{ZX}) was set to .3, .5, and .7. Finally, the population correlation between the operational predictor and criterion (i.e., R_{YZ}) was set to .3 and .5. Thus, the experiment consisted of 36 conditions, three selection ratio conditions and twelve correlational conditions for 3 x 2 x 3 x 2 design.

Procedure

For each of the twelve correlational conditions a dataset was generated to form a population of 1,000,000 cases, each with scores on the experimental predictor (*X*), the operational predictor (*Z*), and the criterion variable (*Y*). Scores on all three variables were standardized with a mean of zero and a standard deviation of 1.0. Within each condition, a random sample of cases was drawn from the population to serve as the applicant sample. From this applicant sample, cases with the top 150 scores on the operational predictor were selected (i.e., top-down selection) for the hired sample, thereby inducing indirect range restriction on the experimental predictor. Thus, the sample size was 100 for all of the restricted correlations. A range-restricted correlation was then computed within each sample. This correlation was corrected using both versions of Thorndike's (1949) Case 3 correction equations, the equation utilizing the restricted predictor intercorrelation (i.e., the more common version) and the equation that calls for the use of the unrestricted predictor intercorrelation (i.e., the less common version). The corrected correlations were then compared to the known population correlation to determine the accuracy of each equation. Finally, a third corrected correlation was generated by correcting the sample correlation via a misapplication of the restricted predictor intercorrelation equation. For this correction, the more common equation, which calls for the restricted correlation between the operational and experimental predictor, was used with the unrestricted predictor intercorrelation. This correction was performed to determine the amount of error caused by a simple misapplication of the equation. Finally, a no range restriction condition in which cases from the applicant sample were randomly selected for the hired sample was created to serve as a baseline for accuracy for the corrected correlations.

In summary, range restricted correlations were corrected three ways: with the restricted predictor intercorrelation equation, with the unrestricted predictor intercorrelation equation, and with a misapplication of the restricted predictor intercorrelation equation in which the unrestricted predictor intercorrelation is employed instead of the restricted value. There was also an uncorrected condition in which cases were randomly selected in order to avoid the effects of range restriction.

For all corrected correlations accuracy was determined by the signed (i.e., raw) difference between the population correlation (i.e., ρ_{XY}) and the various sample correlations corrected for indirect range restriction as well as the squared difference between these values. Results were averaged across 1,000 replications for each condition.

Results

Tables 1-6 list mean bias (i.e., mean error) and mean squared bias (i.e., mean squared error) for the three selection ratios employed in the study for a no range restriction (and thus, no correction) baseline condition, range restriction corrected with the two equations (the restricted predictor intercorrelation and the unrestricted predictor intercorrelation versions), and range restriction corrected with an incorrect application of the more common equation (where the unrestricted predictor intercorrelation is used in place of the restricted value called for by the equation). Tables 7-12 list Cohen's *d* values for the comparison of bias and squared bias for each of the three correction equations to the no range restriction (and thus no correction) condition. These Cohen's *d* values indicate how much more bias and squared bias is present with the range restricted corrected values as compared to a condition where there were no range restriction effects.

Inspection of these tables offers some information regarding the effectiveness of these correction equations. Bias was similar across all selection ratios, although the unrestricted predictor intercorrelation correction equation provided slightly more accurate estimates. As to squared bias, the two equations displayed comparable levels across all selection ratios when the operational and experimental predictors were highly correlated (.7). However, when the selection ratios were low (.1 and .01) and the two predictors were not highly correlated (R_{XZ} = .5 and .3), squared bias was greater when correlations were corrected using the restricted predictor intercorrelation equation than with the unrestricted predictor intercorrelation equation. For example, when the selection ratio was .01 and when R_{XY} = .3, R_{XY} = .5, and R_{XZ} = .3, Cohen's d for the restricted predictor intercorrelation was .635 whereas d for the unrestricted predictor intercorrelation equation was .033. These results provide support for our hypothesis that the unrestricted predictor intercorrelation equation provides more accurate (lower levels of squared bias) estimates than is provided by the restricted predictor intercorrelation equation.

Incorrect usage (inserting the unrestricted predictor intercorrelation into the equation which calls for the restricted correlation) provided less precise estimates across all conditions. This effect was best illustrated when a .01 selection ratio was utilized as Cohen's d values for squared bias ranged from .60 to 1.22. Similar values were also found for bias when incorrect usage occurred. For example, when R_{XY} = .3, R_{XY} = .5, and R_{XZ} = .3, Cohen's d for bias was -1.38. Incorrect usage resulted in less accurate estimates for selection ratios of .1 and .5, as well. In one scenario where R_{XY} = .3, R_{XY} = .5, and R_{XZ} = .7, and SR =.1, Cohen's d for bias was -.75. When SR = .5 in this condition, Cohen's d was -.34. However, across both selection ratios within this same condition, Cohen's d values for both the restricted predictor intercorrelation and unrestricted predictor intercorrelation equations ranged from .13 to .16, highlighting just how inaccurate estimates can be when the correction equation is used in an inaccurate manner. In summary, although there are situations where the bias and squared bias are comparable for correct application of the two equations (e.g., when predictor intercorrelations or selection ratios are high), there is always a price to be paid in accuracy when the traditional equation is used incorrectly.

Discussion

The purpose of this study was to determine if the unrestricted predictor intercorrelation form of the Thorndike (1949) equation correcting for indirect range restriction would provide more accurate estimates (closer to the population correlation) than the restricted predictor intercorrelation form of the equation. Industrial-organizational psychologists routinely use predictive validity studies in order to determine a relationship between selection test scores and later job performance. Indirect range restriction occurs when applicants are selected on the basis of higher scores on the operational predictor, a predictor that is not the focus of the validation study. Researchers and practitioners want the most accurate estimate possible of the unrestricted population correlation to make informed decisions. As mentioned, when selection decisions are made in organizations, the unrestricted predictor intercorrelation will be available if all applicants take both tests (i.e., the operational and experimental predictors) in a predictive design. Therefore, it is of value to know which equation estimates the population correlation with greater accuracy as well as the conditions under which it does so.

Across many of the conditions examined in this study, use of the unrestricted predictor intercorrelation equation over the restricted predictor intercorrelation equation did not affect estimated unrestricted correlation. However, there were some conditions in which the unrestricted predictor intercorrelation equation estimate was decidedly closer to the population correlation than its counterpart. When the correlation between the two predictors and the selection ratio were both low, the unrestricted predictor intercorrelation equation displayed lower levels of squared bias. Additionally, incorrect usage of the traditional correction equation resulted in greater inaccuracy than both the traditional and alternate equation. In fact, incorrect usage frequently resulted in massive errors, with Cohen's *d* values for squared bias as high as 1.21.

It is not surprising that the restricted predictor intercorrelation and the unrestricted predictor intercorrelation forms of the correction equations performed similarly when range restriction was moderate (.5 selection ratio) and diverged in quality when range restriction effects were more extreme (.1 and .01 selection ratios) as the increased levels of range restriction led to larger corrections, and larger corrections magnify errors in the correction. More extreme levels of range restriction are likely to magnify weaknesses inherent to an equation. What is surprising is that for a given selection ratio, lower levels of predictor intercorrelation also demonstrated this pattern of results where the unrestricted predictor intercorrelation equation outperformed the restricted predictor intercorrelation correction equation. Because the restricted predictor intercorrelation and unrestricted predictor intercorrelation versions of the indirect range restriction correction vary only on whether the restricted or unrestricted predictor intercorrelation is used, it might be expected that their results would converge at lower levels of this correlation as the difference between the two values would be at a minimum. Our study found the opposite effect: lower levels of predictor intercorrelation led to the greatest differences between the two equations in terms of squared bias; squared bias was comparable between the two equations at higher levels of predictor intercorrelation.

Conclusion

Researchers and practitioners need guidance for performing adjustments for the effects of range restriction. It is necessary to have the most accurate estimate of the true population correlation to make sound research and business decisions, such as the hiring of employees. The results of this study indicate that it would be beneficial for scholars and practitioners to correct for indirect range restriction using the equation that calls for the unrestricted predictor intercorrelation when that correlation is available.

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Table 1

Mean and Standard Deviation of Bias (.5 Selection Ratio)

Population Correlation ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Baseline (no restriction)		Restricted r_{XZ} Equation		Unrestricted r_{XZ} Equation		Incorrect Usage	
			M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
.3	.3	.3	.007	.075	.004	.083	.005	.079	-.015	.082
.3	.3	.5	.0001	.072	-.002	.087	-.002	.084	-.019	.089
.3	.3	.7	-.0003	.077	.007	.093	.007	.093	-.002	.097
.3	.5	.3	.0004	.072	.0004	.087	-.001	.073	-.045	.076
.3	.5	.5	.006	.077	.006	.09	.004	.081	-.044	.083
.3	.5	.7	-.004	.074	.009	.089	.007	.087	-.031	.088
.5	.3	.3	-.001	.062	.006	.067	.007	.065	-.005	.067
.5	.3	.5	.002	.063	.005	.077	.006	.077	.007	.083
.5	.3	.7	.0007	.063	.004	.087	.004	.087	.018	.094
.5	.5	.3	.001	.062	.004	.07	.004	.059	-.029	.061
.5	.5	.5	.001	.061	.007	.071	.006	.065	-.023	.068
.5	.5	.7	.0006	.061	.005	.071	.005	.07	-.011	.072

Note. N = 1000 for all values.

Table 2

Mean and Standard Deviation of Squared Bias (.5 Selection Ratio)

Population Correlation ρ_{XY} ρ_{ZY} ρ_{XZ}	Baseline (no restriction)		Restricted r_{XZ} Equation		Unrestricted r_{XZ} Equation		Incorrect Usage	
	M	SD	M	SD	M	SD	M	SD
.3 .3 .3	.006	.008	.007	.01	.006	.009	.006	.009
.3 .3 .5	.005	.007	.008	.01	.007	.01	.008	.011
.3 .3 .7	.006	.008	.009	.012	.009	.012	.009	.013
.3 .5 .3	.005	.007	.008	.011	.005	.008	.008	.01
.3 .5 .5	.006	.008	.008	.012	.007	.009	.009	.012
.3 .5 .7	.006	.008	.009	.011	.008	.011	.009	.011
.5 .3 .3	.004	.005	.005	.006	.004	.006	.005	.006
.5 .3 .5	.004	.006	.006	.009	.006	.009	.007	.011
.5 .3 .7	.004	.005	.008	.011	.008	.011	.009	.014
.5 .5 .3	.004	.005	.005	.007	.004	.005	.005	.006
.5 .5 .5	.004	.005	.005	.007	.004	.006	.005	.006
.5 .5 .7	.004	.006	.005	.007	.005	.007	.005	.007

Note. N = 1000 for all values.

Table 3

Mean and Standard Deviation of Bias (.1 Selection Ratio)

Population Correlation ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Baseline (no restriction)		Restricted r_{XZ} Equation		Unrestricted r_{XZ} Equation		Incorrect Usage	
			M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
.3	.3	.3	-.0005	.075	.009	.095	.008	.085	-.033	.11
.3	.3	.5	.003	.072	.012	.11	.012	.104	-.017	.135
.3	.3	.7	.001	.074	.019	.132	.018	.131	.002	.152
.3	.5	.3	-.004	.075	.009	.105	.004	.074	-.099	.087
.3	.5	.5	.002	.074	.004	.107	.003	.087	-.099	.11
.3	.5	.7	.004	.075	.019	.115	.016	.11	-.072	.122
.5	.3	.3	-.0009	.063	.009	.078	.013	.069	-.0009	.095
.5	.3	.5	-.0004	0.06	.013	.09	.017	.09	.034	.125
.5	.3	.7	.004	.064	.003	.108	.005	.11	.044	.133
.5	.5	.3	.001	.064	.007	.085	.007	.057	-.066	.072
.5	.5	.5	.001	.06	.01	.085	.006	.069	-.05	.09
.5	.5	.7	-.0007	.06	.015	.101	.013	.096	-.013	.112

Note. N = 1000 for all values.

Table 4

Mean and Standard Deviation of Squared Bias (.1 Selection Ratio)

Population Correlation ρ_{XY} ρ_{ZY} ρ_{XZ}	Baseline (no restriction)		Restricted r_{XZ} Equation		Unrestricted r_{XZ} Equation		Incorrect Usage	
	M	SD	M	SD	M	SD	M	SD
.3 .3 .3	.006	.008	.009	.012	.007	.01	.013	.016
.3 .3 .5	.005	.008	.012	.017	.011	.016	.012	.026
.3 .3 .7	.005	.008	.018	.026	.018	.027	.023	.034
.3 .5 .3	.006	.008	.011	.015	.005	.008	.018	.018
.3 .5 .5	.006	.008	.011	.016	.008	.011	.021	.023
.3 .5 .7	.006	.008	.014	.02	.012	.019	.02	.023
.5 .3 .3	.004	.005	.006	.009	.005	.008	.009	.014
.5 .3 .5	.004	.005	.008	.013	.008	.014	.017	.027
.5 .3 .7	.004	.005	.012	.018	.012	.019	.02	.033
.5 .5 .3	.004	.006	.007	.01	.003	.005	.009	.01
.5 .5 .5	.004	.005	.007	.01	.005	.007	.011	.013
.5 .5 .7	.004	.005	.01	.016	.009	.016	.013	.019

Note. *N* = 1000 for all values.

Table 5

Mean and Standard Deviation of Bias (.01 Selection Ratio)

Population Correlation			Baseline (no restriction)		Restricted r_{XZ} Equation		Unrestricted r_{XZ} Equation		Incorrect Usage	
ρ_{XY}	ρ_{ZY}	ρ_{XZ}	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
.3	.3	.3	-.0008	.076	-.0009	.114	.009	.09	-.042	.155
.3	.3	.5	-.0007	.074	-.012	.129	-.011	.115	-.05	.181
.3	.3	.7	.002	.076	.04	.154	.035	.158	.019	.199
.3	.5	.3	.0000	.074	.003	.125	.003	.076	-.135	.117
.3	.5	.5	.0007	.077	.044	.131	.021	.106	-.108	.153
.3	.5	.7	.0003	.075	.04	.15	.027	.14	-.096	.168
.5	.3	.3	-.0002	.063	.02	.089	.024	.073	.009	.137
.5	.3	.5	.0009	.061	.012	.107	.02	.109	.07	.179
.5	.3	.7	.004	.062	.024	.134	.032	.143	.106	.19
.5	.5	.3	.004	.063	.002	.102	.01	.059	-.092	.103
.5	.5	.5	-.001	.06	.039	.115	.032	.098	-.007	.163
.5	.5	.7	.003	.062	.04	.129	.035	.126	.005	.164

Note. N = 1000 for all values.

Table 6

Mean and Standard Deviation of Squared Bias (.01 Selection Ratio)

Population Correlation ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Baseline (no restriction)		Restricted r_{XZ} Equation		Unrestricted r_{XZ} Equation		Incorrect Usage	
			M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
.3	.3	.3	.006	.008	.013	.019	.008	.013	.026	.033
.3	.3	.5	.006	.007	.017	.025	.013	.019	.035	.043
.3	.3	.7	.006	.009	.025	.037	.026	.04	.04	.057
.3	.5	.3	.005	.008	.016	.021	.006	.009	.032	.03
.3	.5	.5	.006	.009	.019	.027	.012	.02	.035	.038
.3	.5	.7	.006	.008	.024	.038	.02	.035	.037	.045
.5	.3	.3	.004	.005	.008	.013	.006	.011	.019	.034
.5	.3	.5	.004	.005	.012	.016	.012	.019	.037	.06
.5	.3	.7	.004	.006	.019	.029	.022	.035	.047	.076
.5	.5	.3	.004	.005	.01	.015	.004	.007	.019	.02
.5	.5	.5	.004	.005	.015	.021	.011	.02	.027	.044
.5	.5	.7	.004	.005	.018	.03	.017	.032	.027	.045

Note. $N = 1000$ for all values.

Table 7

Cohen's d for Bias as Compared to Baseline Condition (.5 Selection Ratio)

Population Correlation ρ_{XY} ρ_{ZY} ρ_{XZ}	Restricted r_{XZ} Equation	Unrestricted r_{XZ} Equation	Incorrect Usage
.3 .3 .3	-0.02	-0.02	-0.28
.3 .3 .5	-0.02	-0.03	-0.24
.3 .3 .7	0.09	0.09	-0.02
.3 .5 .3	0.001	-0.02	-0.62
.3 .5 .5	0.002	-0.02	-0.63
.3 .5 .7	0.15	0.13	-0.34
.5 .3 .3	0.11	0.12	-0.06
.5 .3 .5	0.05	0.07	0.07
.5 .3 .7	0.04	0.05	0.22
.5 .5 .3	0.04	0.05	-0.5
.5 .5 .5	0.07	0.07	-0.37
.5 .5 .7	0.07	0.06	-0.18

Note. N = 1000 for all values.

Table 8

Cohen's d for Squared Bias as Compared to Baseline Condition (.5 Selection Ratio)

Population Correlations ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Restricted r_{XZ} Equation	Unrestricted r_{XZ} Equation	Incorrect Usage
.3	.3	.3	0.14	0.07	0.15
.3	.3	.5	0.28	0.24	0.34
.3	.3	.7	0.27	0.26	0.32
.3	.5	.3	0.26	0.03	0.29
.3	.5	.5	0.22	0.08	0.29
.3	.5	.7	0.26	0.22	0.33
.5	.3	.3	0.12	0.07	0.12
.5	.3	.5	0.27	0.25	0.34
.5	.3	.7	0.41	0.42	0.5
.5	.5	.3	0.18	-0.05	0.14
.5	.5	.5	0.21	0.09	0.21
.5	.5	.7	0.2	0.17	0.24

Note. Correction equation estimates were compared to the baseline.

Table 9

Cohen's d for Bias as Compared to Baseline Condition (.1 Selection Ratio)

Population Correlation ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Restricted r_{XZ} Equation	Unrestricted r_{XZ} Equation	Incorrect Usage
.3	.3	.3	0.11	0.1	-0.35
.3	.3	.5	0.1	0.1	-0.19
.3	.3	.7	0.17	0.16	0.004
.3	.5	.3	0.14	0.11	-1.18
.3	.5	.5	0.02	0.01	-1.1
.3	.5	.7	0.16	0.13	-0.75
.5	.3	.3	0.14	0.21	0.005
.5	.3	.5	0.17	0.22	0.35
.5	.3	.7	-0.01	0.02	0.39
.5	.5	.3	0.08	0.1	-0.99
.5	.5	.5	0.12	0.08	-0.067
.5	.5	.7	0.19	0.17	-0.14

Note. N = 1000 for all values.

Table 10

Cohen's d for Squared Bias as Compared to Baseline Condition (.1 Selection Ratio)

Population Correlation ρ_{XY} ρ_{ZY} ρ_{XZ}	Restricted r_{XZ} Equation	Unrestricted r_{XZ} Equation	Incorrect Usage
.3 .3 .3	0.34	0.18	0.59
.3 .3 .5	0.51	0.43	0.68
.3 .3 .7	0.64	0.61	0.71
.3 .5 .3	0.45	-0.02	0.86
.3 .5 .5	0.47	0.21	0.91
.3 .5 .7	0.52	0.47	0.82
.5 .3 .3	0.3	0.14	0.49
.5 .3 .5	0.48	0.46	0.68
.5 .3 .7	0.57	0.57	0.66
.5 .5 .3	0.37	-0.15	0.66
.5 .5 .5	0.46	0.19	0.71
.5 .5 .7	0.55	0.49	0.66

Note. N = 1000 for all values.

Table 11

Cohen's d for Bias as Compared to Baseline Condition (.01 Selection Ratio)

Population Correlation ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Restricted r_{XZ} Equation	Unrestricted r_{XZ} Equation	Incorrect Usage
.3	.3	.3	-0.002	0.116	-0.341
.3	.3	.5	-0.112	-0.103	-0.357
.3	.3	.7	0.313	0.266	0.115
.3	.5	.3	0.033	0.036	-1.383
.3	.5	.5	0.405	0.224	-0.896
.3	.5	.7	0.333	0.238	-0.746
.5	.3	.3	0.264	0.356	0.088
.5	.3	.5	0.123	0.216	0.516
.5	.3	.7	0.194	0.257	0.723
.5	.5	.3	-0.018	0.099	-1.121
.5	.5	.5	0.441	0.411	-0.045
.5	.5	.7	0.362	0.327	0.018

Note. N = 1000 for all values.

Table 12

Cohen's d for Squared Bias as Compared to Baseline Condition (.01 Selection Ratio)

Population Correlation ρ_{XY}	ρ_{ZY}	ρ_{XZ}	Restricted r_{XZ} Equation	Unrestricted r_{XZ} Equation	Incorrect Usage
.3	.3	.3	0.496	0.218	0.828
.3	.3	.5	0.606	0.545	0.957
.3	.3	.7	0.73	0.712	0.842
.3	.5	.3	0.635	0.033	1.22
.3	.5	.5	0.651	0.365	1.05
.3	.5	.7	0.672	0.574	0.976
.5	.3	.3	0.435	0.235	0.604
.5	.3	.5	0.648	0.591	0.769
.5	.3	.7	0.698	0.702	0.803
.5	.5	.3	0.571	-0.07	1.02
.5	.5	.5	0.733	0.48	0.726
.5	.5	.7	0.67	0.574	0.72

Note. N = 1000 for all values.