Spring 2020

Interdependence across Foreign Exchange Rate Markets- A Mixed Copula Approach

Richard Adjei-Boateng
Western Kentucky University, radjeiboateng@gmail.com

INTERDEPENDENCE ACROSS FOREIGN EXCHANGE RATE MARKETS- A MIXED COPULA APPROACH

A Thesis
Presented to
The Faculty of the Department of Mathematics
Western Kentucky University
Bowling Green, Kentucky

In Partial Fulfillment
Of the Requirements for the Degree
Master of Science

By
Richard A. Boateng

May 2020

INTERDEPENDENCE ACROSS FOREIGN EXCHANGE RATE MARKETS – A MIXED COPULA APPROACH

Date Recommended 07/02/2020

Lukun Zheng Digitally signed by Lukun Zheng Date: 2020.07.07 15:52:44 -05'00'

Dr. Lukun Zheng, Director of Thesis

David Zimmer Digitally signed by David Zimmer Date: 2020.07.07 11:45:14 -05'00'

Dr. David Zimmer

Richard Schugart Digitally signed by Richard Schugart Date: 2020.07.02 14:00:08 -05'00'

Dr. Richard Schugart

Ngoc Nguyen Digitally signed by Ngoc Nguyen Date: 2020.07.02 12:30:07 -05'00'

Dr. Ngoc Nguyen

Ranjit T. Koodali Digitally signed by Ranjit T. Koodali Date: 2020.07.08 18:51:04 -05'00'

Associate Provost for Research and Graduate Education

ACKNOWLEDGMENTS

I wish to thank my advisor, Dr. Lukun Zheng for his patience and tactfulness in challenging, inspiring and mentoring me throughout this work.

I am also grateful to the thesis committee members, Dr. David Zimmer, Dr. Ngoc Nguyen and Dr. Richard Schugart for accepting to be on the committee and for their support and roles in making this a success.

My gratitude to the entire faculty of the Mathematics department and the Economics department of Western Kentucky University for their excellent work in making a model university, where I am proud to have gained the experience to make this work possible.

Finally, I thank all my family and friends who have always been there throughout the ups and downs.

CONTENTS

Chapter 1: INTRODUCTION 1

Chapter 2: THE STATISTICAL MODELS 5

2.1 ARMA-GARCH Models for individual exchange rates 5

2.2 Copula 8

2.3 The Mixed Copula Model 11

Chapter 3: ESTIMATION PROCEDURE 18

3.1 Two Stage Estimation 17

3.2 Data Filtration 19

3.3 Estimation of the Empirical CDF and the EM Algorithm 20

Chapter 4: MAIN RESULT 23

4.1 Data 23

4.2 Estimates 23

4.3 Goodness of Fit Test 25

4.4 Empirical Application of the Mixed Copula Model 28

4.5 Diagnostic Test 28

Chapter 5: Conclusion and Future Work 30

BIBLIOGRAPHY 32

LIST OF FIGURES

2.3.1 Gaussian Copula 13

2.3.2 Gumbel Copula 14

2.3.3 Clayton Copula 15

3.2.1 The distribution of $\hat{\alpha}$ when the data are i.i.d. or GARCH (solid line: i.i.d. data; dashed line: GARCH data and filtered; dotted line: GARCH data and unfiltered) 20

3.3.1 Kernel Density and empirical CDF of filtered Euro/Dollar exchange rate returns compared with normal PDF/CDF 21

4.3.1 Mixture of Euro vs Australia Dollars 27

4.5.1 Contour plots of the fitted Mixture copulas overlaid with the contours of the empirical copula for each corresponding exchange rate data set 29

LIST OF TABLES

2.3.1 5th and 95th percentiles for different copula structures 16

3.2.1 Summary Statistics for the ARMA-GARCH model 20

4.1.1 Linear Correlation Coefficient across the four exchange rate markets 23

4.2.1 Results for the estimates for the mixed copula 25

4.3.1 Results for the Cramer Von Mises Statistics 26

INTERDEPENDENCE ACROSS VARIOUS EXCHANGE RATE : A MIXED COPULA APPROACH

Richard Adjei-Boateng May 2020 36 Pages

Directed by: Dr. Lukun Zheng, Dr. David Zimmer, Dr. Richard Schugart and Dr. Ngoc Nguyen

Department of Mathematics Western Kentucky University

The purpose of this thesis is to study the dependence structure of exchange rate pairs using a mixture of copula as opposed to a single copula approach. Mixed copula models have the ability to generate dependence structures that do not belong to existing copula families. The flexibility in choosing component copulas in this mixture model aids the construction of a system that is simultaneously parsimonious and flexible enough to generate most dependence patterns in exchange rate data. Furthermore, the method of mixture copulas facilitates the separation of both the structure and degree of dependence, concepts that are respectively embodied in two essential and distinct parameters for the study of dependence – the weight parameters and the association parameters. The model proposed was constructed to capture various dependence patterns using carefully chosen mixtures of Gaussian, Gumbel and Clayton copulas. We used a two stage semi-parametric approach by first estimating the marginal distributions of each exchange rate pair non-parametrically, and then plugging in the empirical CDF's into the copula. The empirical findings of this experimental study shows a high tendency that each of the exchange rate pairs would either appreciate or depreciate together against the US dollars and that relationship is stronger than that implied by the Gaussian assumption. Our proposed copula mixture model therefore adequately represents the dependence function which appropriately captures the dependence structure between each of the exchange rate pairs in this experimental study. The implications for these findings will be useful for central bank's monetary policies aimed at exchange rate price stabilization as well as for

other stake holders in the exchange rates business. It can also be applied to a wide range of analysis in economics, finance, health, engineering, biology and other related disciplines.

Chapter 1
INTRODUCTION

In simple terms, the value (price) of a country's currency in relation to the currency of another country is referred to as the exchange rate. Modeling the interdependence between exchange rates is of great interest in areas where joint distributions are required and in areas where more than a linear correlation is required. This includes areas of risk management, asset pricing, and portfolio management. In terms of aggregate output, fluctuations in the exchange rates may affect the economy in a way that tend to move prices of imports and the competitiveness of the export industry which in turn partly determines the nature of a country's balance of payment. Central banks use information from this analysis to make important decisions regarding currency interventions. Exchange rates are also used in the valuation of derivatives such as multivariate currency options, and used to hedge against exposure to several currencies in investment portfolio management (Scotti & Benediktsdottir, 2009).

Over the past few years, a number of studies related to economics and finance have investigated the interdependence and contagion of exchange rate series. Yang et al. (2016) finds that three main types of studies focused on the issue of interdependence in foreign exchange markets. An example of the first type is a study by Engle et al. (1988), who contend that exchange rates react not only to shocks in individual markets but also to shocks transmitted across markets. The study used the generalized autoregressive conditional heteroskedasticity (GARCH) model, and since then many papers have discussed the interdependence of exchange rate returns based on the GARCH framework. For example, Pérez-Rodríguez (2006) finds from research based on the dynamic conditional correlation (DCC) GARCH model that, the correlation between the EUR/USD and GBP/USD is particularly high. The second type of study is one that considers the cause-and-effect relationships among different currencies. For example, Spagnolo et al. (2005) provides the causality relationship among forward and spot exchange rates by employing a Markov switching model and instrumental variables. Further, Kenourgios et al. (2011) finds, by applying the vector autoregressive model and Granger causality tests, that the implied volatility of the EUR affects the GBP and the CHF. Beirne & Gieck (2014) also finds that, using a global vector autoregression (VAR) model, the interdependence of foreign exchange markets is notable in developed markets. The third type of study considers non-linear dependence based on the copula functions. For example, Patton (2006), by employing a time-varying copula model provides evidence that the dependence between the DEM/USD and JPY/USD exchange rates is asymmetric. He also finds that the degree of dependence when the currencies depreciate is higher than when they appreciate. Further, Dias & Embrechts (2010) model the dependence of the EUR/USD and the JPY/USD returns based on the copula-GARCH model. They find that a time-varying copula with the proposed interdependence specification gives better results than alternative dynamic benchmark models. According to Patton (2006), asymmetric responses of central banks to exchange rate movements is a possible cause of asymmetric dependence. For example, Patton (2006) explained that a desire to maintain the competitiveness of Japanese exports to the United States with German exports to the United States would lead the Bank of Japan to intervene to ensure a matching depreciation of the yen against the dollar whenever the Deutsche mark (DM) depreciated against the U.S. dollar. On the other hand, a preference for price stability could lead the Bank of Japan to intervene to ensure a matching appreciation of the yen against the dollar whenever the DM appreciated against the U.S. dollar. A distortion of balance in these two objectives could cause asymmetric dependence between these exchange rates. According to Patton (2006), if the competitiveness preference dominates the price stability preference, we would expect the DM and yen to be more dependent during depreciation against the dollar than during appreciation.

The importance of the dependence among different exchange rates cannot be understated, yet they are difficult to predict. This is simply because there are numerous economic and geopolitical factors that affect the exchange rates between two countries especially in a floating regime. They appear to affect each other (Yang et al., 2016). Copulas have been used to study the dependence in many financial and economic time series, (Patton, 2009) and (Patton, 2012). We will investigate the dependency in four exchange rates using a mixed copula approach as opposed to the popular single-copula approach. This is particularly due to the fact that different pairs of exchange rates may show different dependence structures and strengths. Therefore, there is no single copula that is applicable to all situations. By a cautious selection of component copulas in the mixture, we will be able to construct a model that is simultaneously simple and flexible enough to generate most dependence patterns in exchange rate data (Hu, 2006). This thesis is based on the third type of studies, the analysis of the non-linear dependence based on copula functions.

We will analyze the exchange rates of Yen, Euro, Australian dollars and the British Pounds Sterling to the US dollars. We will proceed by estimating the mixture model using a two stage semi-parametric procedure. This procedure will make our estimation robust and free from specification errors since we will first estimate the marginals non-parametrically and then plug in the empirical CDF's into the mixture copula. We will then use the method of Maximum Likelihood (ML) Estimation to estimate the parameters in the mixture copula. The process we use in this paper is based on the assymptotic distribution derived in Genest & Rivest (1993) which shows that under some regularity conditions the ML Estimators in this semi-parametric setup are consistent and asymptotically normal (Hu, 2006). Since most exchange rates data are usually conditional hetroskedastic we will employ the ARMA-GARCH filtering techniques to the exchange rates to take care of possible serial correlation. We will then employ Monte Carlo experiments to study the performance of the ML estimator based on the ARMA-GARCH filtered data. Finally to test for the performance of our model, we will employ the Goodness of Fit test and a Diagnostic test. Our data comprises weekly exchange rates based on the US dollar in relation to the Euro, the Japanese Yen, the Australian dollar and the British Pounds covering a sample period from January 8, 1999 to December 27, 2019 with a total of 1095 observations.

This thesis contribute to the current research in the following ways. First, it compensate for the lack of academic studies on the use of copula mixture models to measure the dependence structure of exchange rates. Second, it examines the dependence structure between the four most traded exchange rates which is the first of its kind. Thus it seeks to provide useful implications for investors and policy makers related to risk management across different regimes. Moreover, understanding the dependence between exchange rates provides meaningful information to carry on a trade.

The structure of the remainder of this thesis as follows: Chapter 2 presents the statistical models; Chapter 3 describes the estimation procedure; Chapter 4 discusses the data and main results; and Chapter 5 concludes the study.

Chapter 2
THE STATISTICAL MODELS

This section describes briefly the statistical models employed in the analysis of this work. It begins by introducing the ARMA-GARCH model for individual exchange rates and then discuss copulas and the mixed copula model.

2.1 The ARMA-GARCH Model for Individual Exchange Rates

The inference procedures utilized in this thesis are based on asymptotic distributions derived for independent and identically distributed (i.i.d) data. However, exchange rate series are usually autocorrelatd and conditional heteroscedastic instead of i.i.d. Hence, before we make inferences about the dependence structure among different exchange rates, we first filter the exchange rate series using an ARMA-GARCH model. In effect researchers adopt the choice of ARMA models for the conditional mean and GARCH models for the conditional variance in time series analysis and modeling (Patton, 2009; Scotti & Benediktsdottir, 2009; Yang et al., 2016). The autoregressive-moving-average model (ARMA) gives a parsimonious description of a typical stochastic process relating two polynomials, the AR (Autoregression) and the MA (Moving averages). Usually, the notation ARMA(r, m) refers to a model with r autoregressive terms (lags) and m moving-average terms (lags). Hence we will first filter the time series using ARMA-GARCH filteration before we conduct any inferential studies. The general ARMA(r, m) component in our model for the conditional mean is expressed as:

$$x_t = c + \sum_{i=1}^{r} \phi_i x_{t-i} + \sum_{j=1}^{m} \theta_i \varepsilon_{t-j} + \varepsilon_t \tag{2.1.1}$$

where,

i. x_t is the time series to be modeled.

ii. c is a constant.

iii. r is the number of autoregressive orders.

iv. m is the number of moving average orders.

v. ϕ_i is the autoregressive coefficients.

vi. θ_j is moving average coefficient.

vii. ε_t is the error. Note that the error term ε_t is also called the innovations.

Note that the deterministic component of Equation (2.1.1), $c+\sum_{i=1}^{r}\phi_i x_{t-i}+\sum_{j=1}^{m}\theta_i\varepsilon_{t-j}$, presents the value of the exchange rate in a current state as function of past observations and errors. The term ε_t, is the random component in Equation (2.1.1). This term is usually considered to have a mean of 0 and constant variance. We will utilize this term for the construction of the GARCH part (variance modeling) of our model and later use it in the copula. However, since investors and policy makers behave differently in different time horizons depending on different times, the error term in the time series for the exchange rates series, ε_t, do not satisfy the homoscedstic assumption of constant variance. In other words, the variance of the error term (ε_t) varies with time and this kind of volatility (heteroskedasticity) which depends on the observations of the immediate past is known as conditional variance. The exchange rate series used in this thesis like many other practical time series will have to be filtered first using ARMA-GARCH model before the asymptotic distribution theories for i.i.d data can be applied. ARCH (auto-regressive conditional heteroskedasticity) models were introduced by Robert Engle to account for this behavior of the error term. Here, the conditional variance process is given an autoregressive structure and the log series are modeled as a white noise multiplied by the volatility, as shown in Equation (2.1.4). In 1986, Tim Bollerslev re-defined the ARCH model to allow it to have an additional autoregressive structure within itself. The GARCH(p, q) part of the model is given by Equation (2.1.3). The conditional variance of the innovations ε_t, σ_t^2 is defined as:

$$\sigma_t^2 = Var_{t-1}(\varepsilon_t) = E_{t-1}(\varepsilon_t^2). \tag{2.1.2}$$

The Generalized Autoregressive Conditional Heteroskedasticity, GARCH (p, q), for the conditional variance of innovations, ε_t, is given by:

$$\sigma_t^2 = k + \sum_{i=1}^{p} G_i \sigma_{t-i}^2 + \sum_{j=1}^{q} A_j \varepsilon_{t-j}^2. \tag{2.1.3}$$

Note that successive innovations are not independent although they are uncorrelated. Actually, the direct generating mechanism for GARCH innovations process, ε_t, is $\varepsilon_t = \sigma_t z_t$, where z_t is a standardized, independent, identically distributed (i.i.d) random variable drawn from some specified probability distribution, usually Gaussian (Pham & Yang, 2010), where σ_t is the conditional standard deviation.

The ARMA-GARCH model can be summarized in the following equations:

$$x_t = c + \sum_{i=1}^{r} \phi_i x_{t-i} + \sum_{j=1}^{m} \theta_i \varepsilon_{t-j} + \varepsilon_t, \tag{2.1.4}$$

$$\varepsilon_t = \sigma_t z_t, \qquad z_t \sim N(0, 1)$$

$$\sigma_t^2 = k + \sum_{i=1}^{p} G_i \sigma_{t-i}^2 + \sum_{j=1}^{q} A_j \varepsilon_{t-j}^2, \tag{2.1.5}$$

where equation (2.1.4) is the ARMA part used to model the mean and Equation (2.1.5) is the GARCH part used to model the variance. For example, the ARMA(1,1)-GARCH(1,1) model can be expressed as:

$$x_t = c + \phi_1 x_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = k + G_1\sigma_{t-1}^2 + A_1\varepsilon_{t-1}^2.$$

In selecting the orders of the ARMA component for each individual exchange rate, the method of Bayesian Information Criterion (BIC) was applied. This method is applied to select the number of lags to be used in the ARMA model. This method selects the model with the lowest BIC. However, for the GARCH component in particular, the simpler GARCH(1,1) model has become widely used in financial time series modeling and is implemented in most statistics and econometric software packages. GARCH(1,1) models are favored over other stochastic volatility models by many economists and mathematicians due to their relatively simple implementation. Also, it is usually sufficient in capturing all of the dependence in the conditional variance, implying that higher order models such as GARCH(4,3) are not necessary (Lumsdaine, 1996; Kat & Heynen, 1994; Lumsdaine, 1995).

2.2 The Copula

A copula is a multivariate function of the marginal distributions which restores the joint distribution among random variables. Technically, it is a multivariate cumulative distribution function for which the marginal probability distribution of each variable is uniform on the interval $[0,1]$. It is used to describe the dependence between random variables. This dependence is important in multivariate studies. The idea of a copula can be dated back to the 19th century when modeling multivariate non-Gaussian distributions was gaining momentum. Modern theories and applications about copula was introduced by Sklar (1959), which states that an n-dimensional joint distribution can be decomposed into its n univariate marginal distributions and an n-dimensional copula (Sklar, 1959).

Theorem 2.1. *[Sklar, 1959] Suppose that H is a distribution function on R^n with marginal distributions $F_1, \ldots, F_n$, then there is a copula C such that*

$$H(x_1, \ldots, x_k) = P\left[F_1(X_1) \leq F(x_1), \ldots, F_n(X_n) \leq F_n(x_n)\right] = C\left[F_1(x_1), \ldots, F_k(x_n)\right] \tag{2.2.1}$$

If H is continuous, then C is unique and is given by

$$C(u_1, \ldots, u_n) = H\left[F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)\right]$$

for $\mathbf{u} = (u_1, \ldots, u_n)^T \in [0,1]^n$ where $\mathbf{x}^T$ is the transpose of the vector of $\mathbf{x}$ and $F_i^{-1}(u_i) = \inf\{x : F_i(x) \geq u\}$, $i = 1, \ldots, n$. Conversely, if C is a copula on $[0,1]^n$ and $F_1, \ldots, F_n$ are distribution functions on R, then the distribution function defined in (2.2.1) is a distribution function on R^n with marginal distributions $F_1, \ldots, F_n$.

As can be seen, a copula maps from a $[0,1]^k$ to $[0,1]$ and connects a k marginal distributions to restore a joint distribution. This means that it models the dependence between individual random variables. Thus, from Theorem 2.1, a copula uses the CDF's (pseudo observations) instead of the actual observations. Because of this, the measurement is invariant under increasing and continuous transformations of the data. This idea is very important when dealing with applied economic data, for example, where the natural logarithm function is usually applied.
This thesis considers bivariate relationships where the symbols x and y $(x, y \in R)$ denotes the observations of random variables of X and Y respectively; and u, v $(u, v \in [0,1])$ to denote their marginal CDFs. We therefore provide a few definitions.

The density(PDF) of a bivariate copula is given by:

$$c(u,v) = \frac{\partial C(u,v)}{\partial u \partial v}. \tag{2.2.2}$$

The density of the bivariate distribution $H(x,y)$ which can be restored by multiplying the copula density with the marginals $f_X(x)$ and $f_Y(y)$ is written as:

$$h(x, y) = c(F_X(x), F_Y(y)) f_X(x) f_Y(y). \tag{2.2.3}$$

A copula $C(u, v)$ exhibits a left tail dependence if,

$$\lim_{u \to 0} \frac{C(u, u)}{u} = \lambda_l > 0. \tag{2.2.4}$$

This tail dependence is very important property of a copula.

A copula will model a right tail dependence if,

$$\lim_{u \to 1} \frac{C(u, u) + 1 - 2u}{1 - u} = \lambda_r > 0. \tag{2.2.5}$$

We introduce the rank correlation statistics called the Kendall's τ. This is a rank correlation which models the relationship between the rankings instead of the actual values of the observations. Hence, it is a robust measure and provides an alternative to linear correlation coefficient for non-elliptical distributions (Hu, 2006). The Kendall's τ is defined as:

$$\tau(X, Y) = P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0], \tag{2.2.6}$$

where $\tau \in [-1, 1]$. A positive τ means positive dependence and a value close to 1 or -1 implies stronger dependence.

The moment condition below gives the relationship between the Kendall's τ and a Copula:

$$\tau = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1. \tag{2.2.7}$$

We now consider how the idea of a single copula can be extended to a mixture of different copulas in the spirit of our model specification.

2.3 The Mixed Copula Model

We make use of three copulas in our mixture model: the Gaussian copula, the Gumbel copula, and the Clayton copula. Gaussian copula has symmetric structure while the other two are asymmetric with Gumbel copula showing heavy right-tail dependence and Clayton copula showing heavy left-tail dependence. Note that the choice of these three copulas allows us to model various dependence structures, which is very common in financial and economic data analysis. A Gaussian copula can be represented as:

$$C_g(u, v; \rho) = \Phi_\rho(\phi^{-1}(u), \phi^{-1}(v)), \tag{2.3.1}$$

where ϕ is the univariate normal distribution and Φ_ρ is the standardized bivariate normal distribution with correlation ρ, ($\rho \in [-1, 1]$). Also, the higher the association parameter ρ, the stronger the dependence. To extend this copula to the multivariate case, we just replace ρ with a correlation matrix. For all elliptical distributions, the following relationship holds:

$$\rho = sin(\frac{\pi}{2}\tau), \tag{2.3.2}$$

where τ is the Kendall's τ and will be discussed later in the chapter.

Figure 2.3.1 shows the scatter plot (top left), contour plot (top right), the cross sectional contour plot (bottom left), and the density (bottom right) of a Gaussian copula with $\rho = 0.5$. Note that the density and contour plots of the copula is based on the definition as expressed in Equation (2.3.1). Also, we observe from the shape of the density plot(bottom right) that with regards to market returns, a Gaussian dependence structure exhibits an equal likelihood for a market to either boom or crash together due to its symmetric nature. The shape of the cross sectional plot (buttom left) indicates a significantly stronger dependence structure during extremes in relation to market returns. These paramount increase in correlations from tranquil periods to volatile periods is called "correlation breakdown" and thus the constancy of correlation overtime between returns such as this cannot be vouched. Boyer et al. (1997) proposes that correlation can barely reveal a thing about the underlying nature of the dependence in such situations. This is because the assumptions for Gaussian distributions themselves requires higher correlations conditional on large co-movements (Patton, 2006). We can make a comparison between the estimated dependence and that of the Gaussian dependence to find out whether there is unusual excess co-movement. For the estimated dependence, when the dependence in volatile periods is higher than that computed in the tranquil periods and when these dependencies are significantly higher than that implied from the Gaussian, then we conclude the presence of contagion. This in effect will be against the Gaussian assumptions.

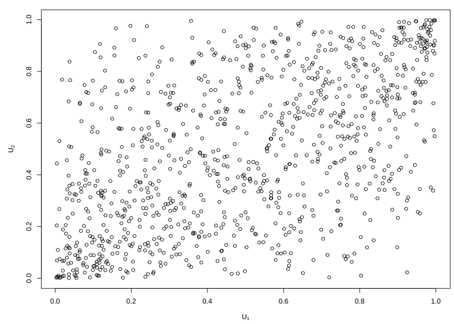
The Gumbel copula is the next copula in our mixture model. It can be represented by the equation:

$$C_m(u, v; \alpha) = \exp\{-[(-log(u))^{\alpha} + (-log(v))^{\alpha}]^{\frac{1}{\alpha}}\}, \tag{2.3.3}$$

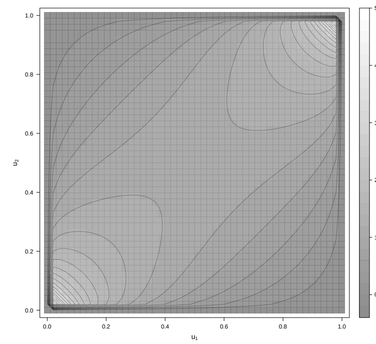
Here, the association parameter is α, and $\alpha \in [1, \infty)$. The relationship between α and the strength of dependence is positive. We have:

$$\alpha = \frac{1}{1 - \tau} \tag{2.3.4}$$

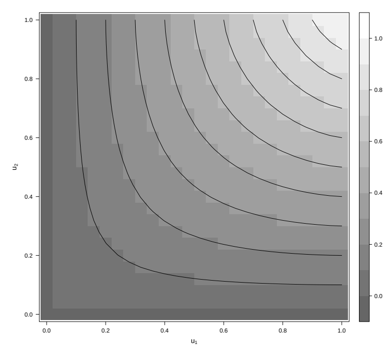
which can be derived from Equation (2.2.7). Figure 2.3.2 shows the scatter plot(top left), contour plot(top right), cross sectional plot(buttom left) and the density plot(buttom right) of the gumbel copula. We observe that the Gumbel copula is asymmetric about



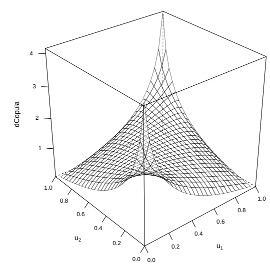
(a) scatter plot



(b) contour plot



(c) cross-sectional plot

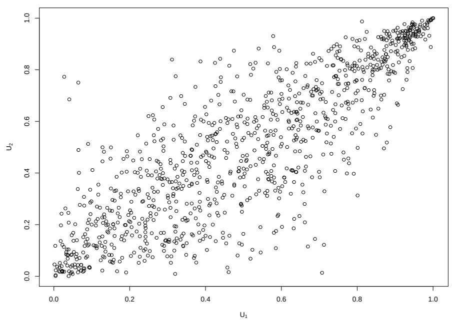


(d) density plot

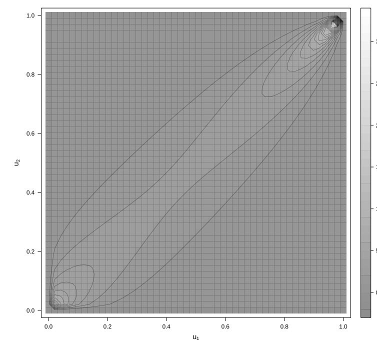
Figure 2.3.1: The (a) scatter plot ,(b) contour plot, (c) cross sectional plot and (d) the density plot of a Gaussian Copula with $\rho = 0.5$.

(1/2, 1/2) and more density is put on the right tail. Using Equations (2.2.4) and (2.2.5) we can easily show that the Gumbel copula is right tailed dependence . That is $\lambda_l = 0$ and $\lambda_r = 2 - 2^{\frac{1}{\alpha}}$. Also, the shape of the density (buttom right) of the copula from Figure 2.3.2 implies that if two markets are more likely to boom together rather than crash together in terms of market returns, this copula will perform well in capturing such dependence.

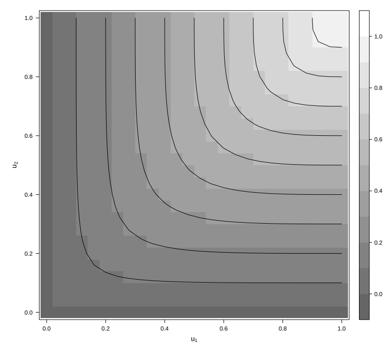
The third copula function in the mixture is the Clayton copula which is defined as:



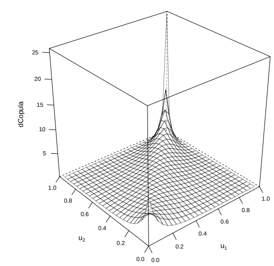
(a) scatter plot



(b) contour plot



(c) cross sectional plot



(d) density plot

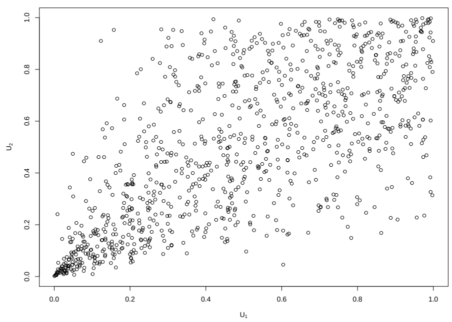
Figure 2.3.2: The (a) scatter plot, (b) contour plot, (c) cross sectional plot and (d) the density plot of a Gumbel Copula with $\alpha = 0.5$.

$$C_c(u, v; \beta) = max([u^{-\beta} + v^{-\beta} - 1]^{\frac{-1}{\beta}}, 0) \tag{2.3.5}$$

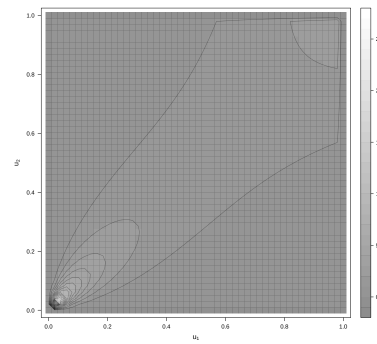
where $\beta \in [-1, \infty)\backslash\{0\}$.

The relationship between the parameter β, of Clayton copula and Kendall's τ is $\beta = \frac{2\tau}{1-\tau}$.

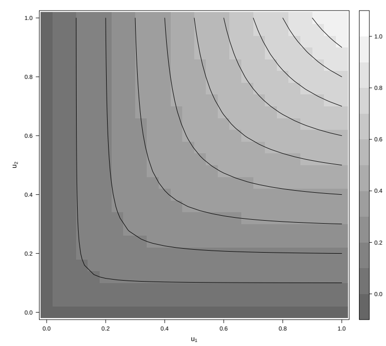
The characteristics of the Clayton copula is equivalent to the opposite of the Gumbel copula. It captures the left tail dependence. Precisely, using Equations (2.2.4) and (2.2.5), we can show that the Clayton copula has a positive left tail dependence. That is, $\lambda_r = 0$ and $\lambda_l = 2^{\frac{-1}{\beta}}$. Figure 2.3.3 shows the scatter plot(top left),



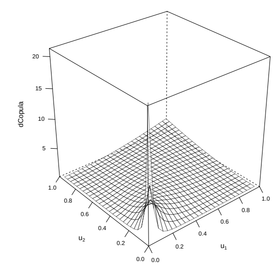
(a) scatter plot



(b) contour plot



(c) cross sectional plot



(d) density plot

Figure 2.3.3: The (a) scatter plot, (b) contour plot, (c) cross sectional plot and (d) the density plot of a Clayton Copula with $\beta = 0.5$.

contour plot(top right), cross-sectional plot(buttom left) and the density plot(buttom right) of the Clayton copula. The shape of the density plot(buttom right) implies that when two markets are likely to crash together than boom together, this copula will do a better job of modeling such a situation.

Below is a practical example showing the difference between the three dependence structures used in this research. If we take a Kendall's τ between two exchange rates series to be 0.2, then from Equations (2.3.2) and (2.3.4), we can compute $\rho = 0.31$, $\alpha = 0.8$ and $\beta = 0.5$ for the Gaussian, Gumbel and Clayton copulas respectively. Therefore, the probability that two pairs of exchange rates are in their lowest 5th percentiles and their highest 95th percentiles are given in Table 2.3.1. For the purpose of illustration, we add the percentiles of an independent copula. As we expect, the probability that both exchange rates returns hits their lowest 5th and highest 95th percentiles for the Gaussian copula are both equal to 0.0073. This is due to its symmetric nature. However, when a Gumbel and Clayton copulas are used to measure the dependence, we realize the probability that the two exchange rates are in their lowest 5th percentile decreases to 0.0054 and then increases to 0.0158 for the Clayton copula. Clayton copula is therefore seen to be astute in capturing left tail dependence. Note that when we ignore any correlation between the two exchange rates, as can be seen from the last column of the table, the results would have led to a significant underestimation of the joint dependence between the exchange rates pair. Similarly, we see the probability that two markets are in their highest 95th percentile as measured using the Gumbel copula is 0.0146, which is greater than both the Gaussian and Clayton copulas probability estimates. Thus, the Gumbel copula is suitable for measuring right tail dependence. We realize again that ignoring the conditional dependence between the two exchange rate pairs would have underestimated the results in terms of the probability, as can be seen from the last column of Table 2.3.1.

quantiles	Gussian	Gumbel	Clayton	Independent
5%	0.0073	0.0054	0.0158	0.0025
95%	0.0073	0.0146	0.0037	0.0025

Table 2.3.1: Percentiles for different copula structures.

By implication, we can infer that the probability that both markets are in their highest 95th percentile is 0.0073 or about 7 times in a millennium. However, when we assume the Gumbel copula, the probability of such an event is 0.0146, which is about 15 times in a millennium, more than twice as likely than under the Gaussian assumption.

We will define the mixture of copula model using these three different copulas as follows:

$$C_{mix}(u, v; \rho, \alpha, \beta, w_1, w_2) = w_1 C_g(u, v; \rho) + w_2 C_m(u, v; \alpha) + (1 - w_1 - w_2) C_c(u, v; \beta) \tag{2.3.6}$$

where,

i. w_1 and w_2 are both between 0 and 1 and $w_1 + w_2 \leq 1$. Also, w_1 and w_2 are the weight parameters that reflect the dependence structures.

ii. ρ, α and β are the association parameters (degree of dependence).

Chapter 3

ESTIMATION PROCEDURE

3.1 The Two-Stage Estimation

To estimate the parameters in the copula model, there are generally two main approaches. In the first approach, one may estimate the copula and the marginal distributions simultaneously. Utilizing the method of Maximum likelihood estimator (MLE), one can easily compute the likelihood using the density from Equation (2.2.3). This approach requires that the parameters in the marginals and that in the copula are estimated jointly, thus it may lead to complex computational issues. In addition, this requires that the marginal distributions be explicitly specified and any error in the specification step will be detrimental to the estimation of the copula. In the second approach, one can estimate the model in two stages. The first stage requires that we estimate the marginals assuming that the two random variables are independent. In the second stage, we plug the estimated marginal functions into the copula and use MLE to estimate the parameters in the copula. This thesis makes use of the second approach in our estimations. That is estimating the marginals non-parametrically and then plugging in the empirical CDF's into the copula. Since we do not have to specify the marginals, our estimation will be robust and will avoid errors due to the marginal specification issues.

In short, to estimate the mixed copula between two data series $\{X_t\}_{t=1}^n$ and $\{Y_t\}_{t=1}^n$,

i. First filter the original data with ARMA-GARCH model.

ii. Second, compute the empirical CDFs of each filtered series assuming independence.

iii. Finally, use EM algorithm to implement the estimation.

3.2 Data Filtration

We apply the asymptotic theories presented in Genest et al. (1995) for i.i.d. data which shows that the ML estimators in this semi-parametric setup are consistent and asymptotically normal. But, economic and financial data are far from i.i.d. They are usually conditional heteroskedastic. We solve this problem by filtering our data using the ARMA-GARCH filter. In Table 3.2.1, we report the estimates, with standard errors in parentheses, of the ARMA-GARCH parameters for the four exchange rates as defined in Equations 2.1.4 and 2.1.5.

We further conduct an experiment to show the validity of the filtration. We make use of the Monte Carlo simulations to implement this experiment. During this experiment, we compare three MLE estimators for i.i.d. data, filtered GARCH data, and unfiltered GARCH data. The study is conducted on a Gumbel copula with true value of parameter $\alpha = 2.0$. We set the number of iterations to be 1000 and the sample size to be 2000. We let $\hat{\alpha}$ denote the ML estimator of α. In Figure 3.2.1, the solid line(black) shows the density of $\hat{\alpha}$ when the data is i.i.d.; the dashed line(red) shows the density of $\hat{\alpha}$ when the data is filtered and generated by GARCH(1, 1) and; the dotted line(blue) shows the density of $\hat{\alpha}$ when the data is unfiltered and generated by GARCH(1, 1). We observe that the estimator for filtered GARCH data (mean = 2.0009) closely resemble the estimator for i.i.d. data (mean = 2.0019) but, when the data is unfiltered and GARCH (mean = 1.8454), we observe the estimator is biased to the left. This implies that, whenever we have conditional heteroscedastic data, large volatility clustering results in underestimation of the degree of dependence.

Exchange Rates	ARMA Orders	ARMA Constants	ARMA Coefficient	GARCH Constant	Lagged Variance	lagged ε^2
AUS/US	(0, 1)	0.000089	0.267745	0.000007	0.103536	0.853525
		(0.000424)	(0.031638)	(0.000001)	(0.012515)	(0.015427)
Euro/US	(0, 1)	0.000030	0.260660	0.000002	0.073061	0.914051
		(0.000327)	(0.030981)	(0.000010)	(0.121006)	(0.134039)
Pounds/US	(1, 1)	-0.000019	-0.032463,0.313104	0.000009	0.132161	0.782916
		(0.000347)	(0.105035),(0.098929)	(0)	(0.013670)	(0.018729)
Yen/US	(0,1)	0.000014	0.271058	0.000005	0.075117	0.884538
		(0.000378)	(0.030816)	(0.000001)	(0.008949)	(0.013728)

Table 3.2.1: Summary for the ARMA-GARCH specifications. ARMA orders and the corresponding parameters were selected based on BIC for data filtration in the exchange rate time series.

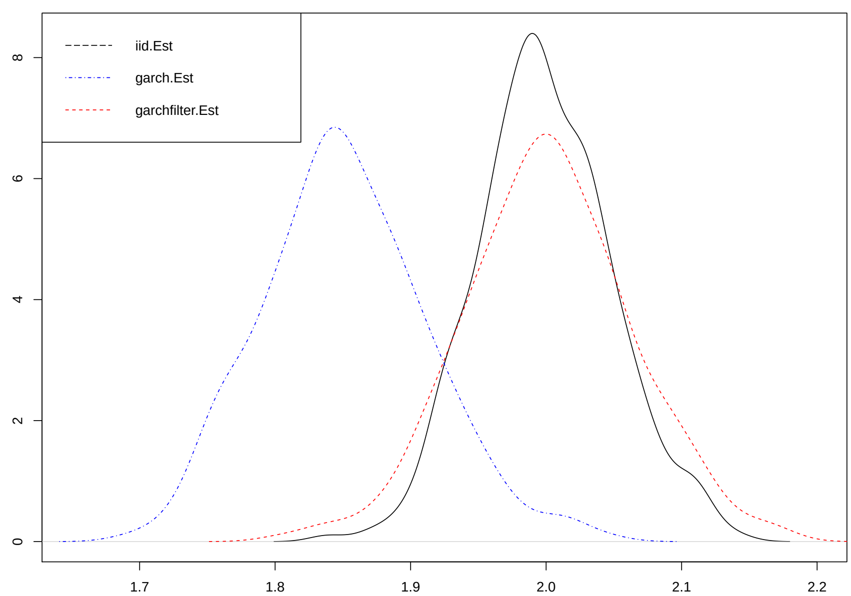


Figure 3.2.1: The distribution of $\hat{\alpha}$ when the data are i.i.d (black solid line), GARCH and unfiltered (blue dotted line) and GARCH and filtered (red dotted line).

3.3 Estimation of the Empirical CDF and the EM Algorithm

As described in the section above, the time series data was filtered with the ARMA-GARCH model. We then compute the empirical CDF's of the filtered data.

In Figure 3.3.1, the graph on the left shows the kernel density (solid line) of the filtered Euro/Dollar exchange rate and the corresponding normal density (dashed line). Note that estimating the marginals parametrically would require the features of the marginals be specified explicitly. The plot on the right of Figure 3.3.1 compares the empirical CDF with a normal probability distribution function.

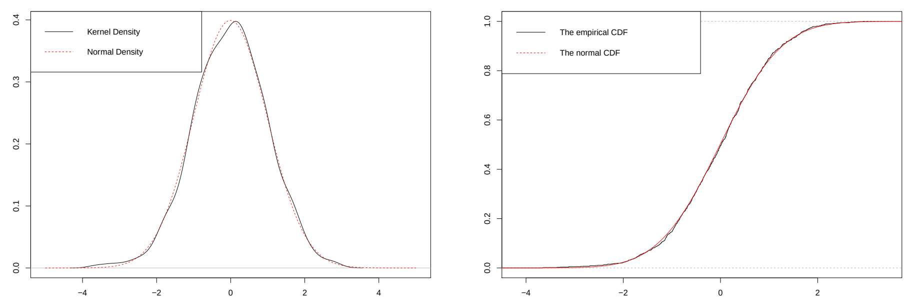


Figure 3.3.1: Kernel Density (solid black line) and empirical CDF of filtered Euro/Dollar exchange rate returns compared with normal PDF/CDF (red dotted line)

To implement the estimation described above, we utilize EM algorithm. Primarily, researchers use the EM algorithm's design to solve Maximum likelihood problems that emanates from incomplete data. With a little modification, we treat the information that each observation is drawn from which distribution as missing data in the mixture model.

In other words, we estimate the marginals non-parametrically and then plug the empirical CDFs into the copula. This approach is advantageous in that we do not need to specify the marginals. Therefore we will have a robust and error free specification estimations for the marginals. For instance if we generate a sample $(z_1, z_2, ..., z_n)$ independently from a univariate distribution $F_Z(z)$, the empirical CDF(used to form the sample) of Z can be computed using:

$$\hat{F}_Z(z) = \frac{1}{n}\sum_{t=1}^{n} 1\{Z_t \leq z\} \tag{3.3.1}$$

If $\hat{F}_X(x)$ and $\hat{F}_Y(y)$ represents the empirical CDF's of X and Y, then we can write the joint distribution as

$$\hat{H}(x, y; \theta) = C(\hat{F}_X(x), \hat{F}_Y(y); \theta),$$

where θ denotes the parameters in the copula.

Note that the other exchange rates which will be used in the study will follow similar form using the ARMA-GARCH models. We will name the above specifications the "copula models" for the marginal distributions, because they will be used with the copula models introduced above.

Chapter 4

MAIN RESULTS

4.1 Data

Our data comprises weekly exchange rates based on the Euro, the Australian dollar and the British pound sterling and the Japanese Yen per US dollar exchange rates, covering a sample period from January 8, 1999 to December 27, 2019. The data set was retrieved from FRED, Federal Reserve Bank of St. Louis website. Our sample size is 1095. We present the correlation coefficient matrix between the four data sets in Table 4.1.1. We observe that the strongest correlation rate is the Australia vs Euro pair whereas the weakest correlation pair is the Pounds vs Yen.

	Euro	GBP	JPY
AUS	0.827	0.278	0.752
Euro		0.549	0.546
GBP			-0.001

Table 4.1.1: Linear correlation coefficient across the four exchange rates.

4.2 Estimates

Table 4.2.1 shows the estimated parameters in the mixture model. We come to the following conclusions. First, the model exhibits significant association parameters and shape parameters across the three copula mixture. This implies that the dependency shown by the exchange rate pairs exhibits both lower and upper tail dependence. Therefore all the data pairs under consideration will poorly be modeled under only the Gaussian assumptions. In other words, there is a high tendency that two exchange rates markets would either appreciate or depreciate together against the US dollars and that relationship is stronger than implied by just a single copula such as the Gaussian copula assumption. Furthermore, we realize that the AUS vs Yen pair and the Pounds vs Yen pair among the six pairs exhibits a negative Gaussian association parameter ρ (-0.0118 and -0.1260 respectively). A small negative association Gaussian parameter ρ implies a weak negative dependence during tranquil periods, when these exchange rates are in their normal "levels" of range against the US dollars. However, a high association parameters for the Gumbel and Clayton copulas for the exchange rate implies a stronger tail dependence, that is when they are rigorously rising against the US dollars or falling against the US dollars. Thus, we find that the dependency on both the right tail and the left tail are much stronger than that implied by only the Gaussian assumption. However, the dependency during the tranquil periods are similar to that implied from a bi-variate normal distribution. This implies that the exchange rate pair under consideration exhibits both left and right tail dependence and cannot be adequately modeled by just the Gaussian assumption of dependence alone. In other words, the tendency that two exchange rate markets boom and fall together is stronger than implied by a Gaussian dependence structure. Overall, we find that all the exchange rate pairs exhibit asymmetric dependence structure. This behavior of asymmetric dependency of exchange rates is in line with the findings of Patton (2006) on the study of the DM-dollar (Euro Dollar) and Yen-dollar exchange rates. They found that these asymmetric dependency of exchange rates is due to the behavior of the actions and reactions of the central banks in response to exchange rate movements. For instance, the desire to be competitive enough would lead the bank of Australia to intervene to ensure a matching depreciation of the British pounds sterling against the US dollars whenever the pounds depreciated against the US dollars and generate a stronger dependency during a depreciation of the pounds and Australian dollars against the US dollars. This behavior of the exchange rates markets is similar to what is popularly known in the financial market as "contagion". There is a large literature about theoretical modeling of the market co-movement in which the general idea about financial contagion is seen to be explained by financial links including herding behavior and fear based spillover (Allen & Gale, 2000), and by real links including trading and production (Forbes & Warnock, 2012), and by political factors.

	AUS Euro	AUS Pounds	AUS Yen	Euro Pounds	Euro Yen	Pounds Yen
Gaussian($\hat{\rho}$)	W:0.28,0.7163	W:0.31, 0.6584	W:0.38, -0.0118	W:0.45, 0.7128	W:0.37, 0.6155	W:0.39, -0.1260
	(0.02)	(0.04)	(0.30)	(0.01)	(0.07)	(0.03)
Gumbel($\hat{\alpha}$)	W:0.35, 1.7008	W: 0.42,1.3252	W:0.30, 1.1758	W: 0.33, 1.7960	W:0.36, 1.0416	W: 0.33,1.2516
	(0.04)	(0.03)	(0.10)	(0.04)	(0.04)	(0.04)
Clayton($\hat{\beta}$)	W:0.37,0.5721	W:0.28 , 1.2907	W:0.32,0.8600	W:0.22, 1.7916	W:0.27, 0.2299	W:0.28, 0.6717
	(0.03)	(0.07)	(.50)	(0.06)	(0.18)	(0.12)

Table 4.2.1: Estimates for the mixed copula

4.3 Goodness of Fit Test

To to evaluate the performance of the estimated mixture model, we perform the goodness of fit test in this section. The relevance of this section is to verify whether the proposed mixture model satisfactorily models the dependence between the four exchange rates under consideration. We consider all the six pairs of exchange rate data and our approach is similar to the one conducted by Genest & Rémillard (2008). The test statistic called the Cramer Von-Mises statistic can be expressed as

$$S_n^{gof} = \sum_{i=1}^{n} (C_n(U_{i,n})) - C_{\theta_n}(U_{i,n}))^2 \tag{4.3.1}$$

where,

i. S_n^{gof} is the Cramer Von-Mises Statistic.

ii. C_n is the empirical copula.

iii. $U_{i,n}$ is the Pseudo observations from the estimated margins.

iv. C_{θ_n} is the estimated mixture copula.

v. n is the sample size of the Pseudo observation.

An approximate p-value for the test can be obtained by means of a parametric bootstrap whose asymptotic soundness is investigated by Genest & Rémillard (2008).

The Goodness of fit test amounts to formally testing that

H_0 : C_{mixed} fits the data well and

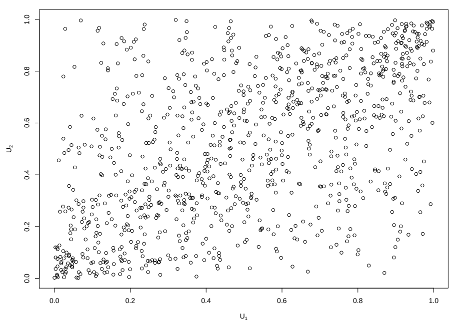
H_A : C_{mixed} does not fit the data well,

where C_{mixed} is our estimated mixture copula. The empirical copula is a consistent estimator of the unknown copula C whether H_0 is true or not, as suggested in Fermanian et al. (2005) and Genest & Rémillard (2008). This goodness-of-fit test consists of comparing C_n with an estimate C_{θ_n} of C obtained under the assumption that $C \in \zeta$ is true. Note that θ_n is an estimate of θ computed from Equation 3.3.1. Thus, we considered taking the Cramer Von-Miss Statistic for all the six exchange rate pairs and their corresponding p-values using bootstrap simulation with $N = 500$ (Hofert et al., 2019).

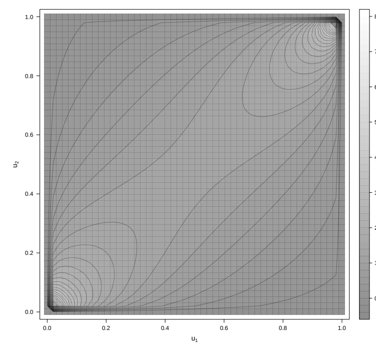
	Exchange_pairs	CM_statistics	P_values
1	AUS/EURO	0.01	0.51
2	AUS/POUNDS	0.01	0.93
3	AUS/YEN	0.03	0.06
4	EURO/POUNDS	0.01	0.42
5	EURO/YEN	0.03	0.08
6	POUNDS/YEN	0.02	0.16

Table 4.3.1: Cramer Von Mises Statistics

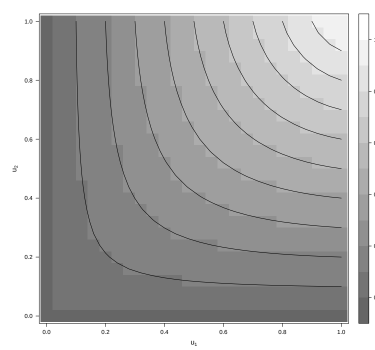
Table 4.3.1 shows the cramer Von-Mises statistics for each data pair and the corresponding p-values. We realize from the table that the goodness of fit statistic based on the Cramer Von-Mis statistic all lies outside the critical region with p-values greater than the 0.05. In other words, we fail to reject the null hypothesis that our mixture copula model fits the data well for all the six pairs.



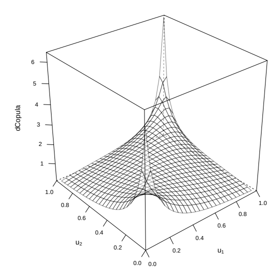
(a) scatter plot



(b) contour plot



(c) cross sectional plot



(d) density plot

Figure 4.3.1: The (a) scatter plot, (b) contour plot, (c) cross sectional plot and (d) the density plot of a Mixture of Euro vs Australia Dollars exchange rates with $\rho = 0.7163$, $\alpha = 1.7008$ and $\beta = 0.5721$ and weights 0.28, 0.35 and 0.37 for the Gaussian, Gumbel and Clayton copulas respectively.

We take the model for the Euro and Australia dollar pair mixture model as an example. As shown in Figure 4.3.1 these dependence structures can be visualized by making a scatter plots, contour plots, cross sectional plots and a density plots. Note that, unlike its components, the constructed mixture has both lower and upper tail dependence with different weights on the component copulas.

4.4 Empirical Application of the Mixed Copula Model

Reconstruction of a joint distribution of a direct statistical application of an estimated mixed copula model. It has been shown in the previous sections that the dependence structures between economic and financial markets exhibits L and U shapes. Their marginal distributions are also characterized by left skewness with thick tails (see Figure 3.2.1). Hence it is better to specify their joint distribution using mixed copulas. Specifying their joint distribution with only existing families becomes difficult. The process of reconstruction using mixed copulas involves two cogent steps:

i. First we estimate the copula with non-parametrically estimated marginals.

ii. Second, specify the marginals and plug them into the estimated copula function.

If we take the Euro and Australian dollar pair as an example, we have

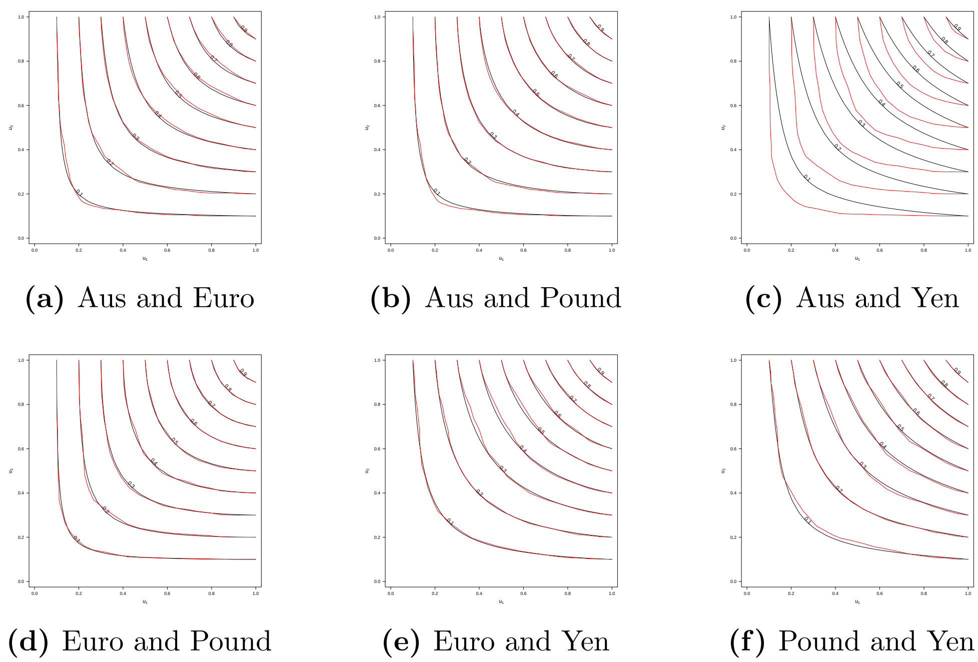
$$H(X,Y;\gamma_x,\gamma_y) = 0.28C_g[F_X(x;\gamma_x),F_Y(y;\gamma_y);0.7163] +$$
$$0.35C_m[F_X(x;\gamma_x),F_Y(y;\gamma_y);1.7008] + 0.37C_c[F_X(x;\gamma_x),F_Y(y;\gamma_y);0.5721]$$

,

where γ_x and γ_y are the parameters in the marginal distributions of X and Y respectively. The dependence function and the marginals are estimated separately. The copula models the dependence structures and we specify the marginals separately. This model approach is able to provide a more realistic description of the data generating process compared to just a joint Gaussian model.

4.5 Diagnostic Tests

We perform a graphical diagnostic test to assess which estimated copula fits best. This involve comparing the contour plots of the parametric estimates with those of the empirical copula for each of the corresponding exchange rate data set. In Figure 4.5.1, we plot the empirical copula (red line) and parametric estimates (black line) for each of the six pairs of markets. As can be seen from the graphs, the models predictions for all the six pairs closely matches and suggest that the mixture model fits better.



(a) Aus and Euro **(b)** Aus and Pound **(c)** Aus and Yen

(d) Euro and Pound **(e)** Euro and Yen **(f)** Pound and Yen

Figure 4.5.1: Contour plots of the fitted Mixture copulas (black line) overlaid with the contours of the empirical copula (red line) for each corresponding exchange rate data set.

Chapter 5

Conclusion and Future Work

We have so far showed that the estimation of dependence is very important in economics and finance. In addition, we showed that using a mixture of copula models to empirically measure cross-market dependence presents many flexibility. This flexibility is mainly due to the structure of the mixture copula itself. The degree of dependence is carried through the association parameters and the structure of the dependence is summarized by the weight of each individual copula function. We proceeded by using a semi-parametric approach for the model estimation and computed the dependence structures in the underlined data set. We found that all the pairs of exchange rates under the study exhibited positive weight on all the three copulas in the mixture. This implies that the dependency shown by the exchange rate pairs exhibits both lower and upper tail dependence. Therefore all the data pairs under consideration will poorly be modeled under only the Gaussian assumptions. Furthermore, the AUS vs Yen pair and the Pounds/Yen pair among the six pairs exhibits a negative association parameter ρ (-0.0118 and -0.1260 respectively) for the Gaussian component. A small negative association Gaussian parameter ρ implies a weak negative dependence during tranquil periods , that is when these exchange rates are in their normal "levels" of range against the US dollars. However, a high association parameters for the Gumbel and Clayton copulas for the exchange rate implies a stronger tail dependence, that is when they are rigorously rising against the US dollars or falling against the US dollars.

Our findings also emphasized that exchange rate pairs that have lower correlations such as the Pounds and Yen pairs have almost the same probability to crash together as pairs that have higher correlation coefficient such as the Euro and Australia pair. We realized this situation can be explained by the different estimated weight parameters in the mixed copula model.

We expect future research to gravitate towards comparing different tail dependence structures such as the symmetrized Joe Clayton copula (Joe-Clayton copula conditioned to capture both upper and lower tail dependence) to selected mixed copula models, targeting their relative performance over exchange rate data.

References

Allen, F., & Gale, D. 2000. Financial contagion. *Journal of political economy*, *108*(1), 1–33.

Beirne, J., & Gieck, J. 2014. Interdependence and contagion in global asset markets. *Review of International Economics*, *22*(4), 639–659.

Bhatti, M. I., & Do, H. Q. 2019. Recent development in copula and its applications to the energy, forestry and environmental sciences. *International Journal of Hydrogen Energy*.

Boyer, B. H., Gibson, M. S., Loretan, M., et al. 1997. *Pitfalls in tests for changes in correlations* (Vol. 597). Board of Governors of the Federal Reserve System Washington, DC.

Chen, X., & Fan, Y. 2006. Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspecification. *Journal of econometrics*, *135*(1-2), 125–154.

Dias, A., & Embrechts, P. 2010. Modeling exchange rate dependence dynamics at different time horizons. *Journal of International Money and Finance*, *29*(8), 1687–1705.

Dong, H., Xu, X., Sui, H., Xu, F., & Liu, J. 2017. Copula-based joint statistical model for polarimetric features and its application in polsar image classification. *IEEE Transactions on Geoscience and Remote Sensing*, *55*(10), 5777–5789.

Elie, B., Naji, J., Dutta, A., & Uddin, G. S. 2019. Gold and crude oil as safe-haven assets for clean energy stock indices: Blended copulas approach. *Energy*, *178*, 544–553.

Ene, L. T., Næsset, E., & Gobakken, T. 2013. Model-based inference for k-nearest neighbours predictions using a canonical vine copula. *Scandinavian journal of forest research*, *28*(3), 266–281.

Engle, R. F., Ito, T., & Lin, W.-L. 1988. *Meteor showers or heat waves? heteroskedastic intra-daily volatility in the foreign exchange market* (Tech. Rep.). National Bureau of Economic Research.

Erb, C. B., Harvey, C. R., & Viskanta, T. E. 1994. Forecasting international equity correlations. *Financial analysts journal*, *50*(6), 32–45.

Fermanian, J.-D., et al. 2005. Goodness-of-fit tests for copulas. *Journal of multivariate analysis*, *95*(1), 119–152.

Forbes, K. J., & Warnock, F. E. 2012. Capital flow waves: Surges, stops, flight, and retrenchment. *Journal of international economics*, *88*(2), 235–251.

Frees, E. W., Carriere, J., & Valdez, E. 1996. Annuity valuation with dependent mortality. *Journal of Risk and Insurance*, 229–261.

Genest, C., Ghoudi, K., & Rivest, L.-P. 1995. A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*, *82*(3), 543–552.

Genest, C., & Rémillard, B. 2008. Validity of the parametric bootstrap for goodness-of-fit testing in semiparametric models. In *Annales de l'ihp probabilités et statistiques* (Vol. 44, pp. 1096–1127).

Genest, C., & Rivest, L.-P. 1993. Statistical inference procedures for bivariate archimedean copulas. *Journal of the American statistical Association*, *88*(423), 1034–1043.

Hofert, M., Kojadinovic, I., Mächler, M., & Yan, J. 2019. *Elements of copula modeling with r*. Springer.

Hu, L. 2006. Dependence patterns across financial markets: a mixed copula approach. *Applied financial economics*, *16*(10), 717–729.

Inagaki, K. 2007. Testing for volatility spillover between the british pound and the euro. *Research in International Business and Finance*, *21*(2), 161–174.

Jiang, Y., Lao, J., Mo, B., & Nie, H. 2018. Dynamic linkages among global oil market, agricultural raw material markets and metal markets: An application of wavelet and copula approaches. *Physica A: Statistical Mechanics and its Applications*, *508*, 265–279.

Jung, R. C., & Maderitsch, R. 2014. Structural breaks in volatility spillovers between international financial markets: Contagion or mere interdependence? *Journal of Banking & Finance*, *47*, 331–342.

Kangas, A., Myllymäki, M., Gobakken, T., & Næsset, E. 2016. Model-assisted forest inventory with parametric, semiparametric, and nonparametric models. *Canadian Journal of Forest Research*, *46*(6), 855–868.

Kat, H. M., & Heynen, R. C. 1994. Volatility prediction: A comparison of the stochastic volatility, garch (1, 1) and egarch (1, 1) models. *Journal of Derivatives*, *2*(2).

Kenourgios, D., Samitas, A., & Paltalidis, N. 2011. Financial crises and stock market contagion in a multivariate time-varying asymmetric framework. *Journal of International Financial Markets, Institutions and Money*, *21*(1), 92–106.

Kershaw, J. A., Weiskittel, A. R., Lavigne, M. B., & McGarrigle, E. 2017. An imputation/copula-based stochastic individual tree growth model for mixed species acadian forests: a case study using the nova scotia permanent sample plot network. *Forest Ecosystems*, *4*(1), 1–13.

Klein, B., Meissner, D., Kobialka, H.-U., & Reggiani, P. 2016. Predictive uncertainty estimation of hydrological multi-model ensembles using pair-copula construction. *Water*, *8*(4), 125.

Li, D. X. 2000. On default correlation: A copula function approach. *The Journal of Fixed Income*, *9*(4), 43–54.

Li, J., & Li, P. 2019. Empirical analysis of the dynamic dependence between wti oil and chinese energy stocks. *Energy Economics*, 104299.

Lumsdaine, R. L. 1995. Finite-sample properties of the maximum likelihood estimator in garch (1, 1) and igarch (1, 1) models: A monte carlo investigation. *Journal of Business & Economic Statistics*, *13*(1), 1–10.

Lumsdaine, R. L. 1996. Consistency and asymptotic normality of the quasi-maximum likelihood estimator in igarch (1, 1) and covariance stationary garch (1, 1) models. *Econometrica: Journal of the Econometric Society*, 575–596.

Patton, A. J. 2006. Modelling asymmetric exchange rate dependence. *International economic review*, *47*(2), 527–556.

Patton, A. J. 2009. Copula–based models for financial time series. In *Handbook of financial time series* (pp. 767–785). Springer.

Patton, A. J. 2012. A review of copula models for economic time series. *Journal of Multivariate Analysis*, *110*, 4–18.

Pérez-Rodríguez, J. V. 2006. The euro and other major currencies floating against the us dollar. *Atlantic Economic Journal*, *34*(4), 367–384.

Pham, H. T., & Yang, B.-S. 2010. Estimation and forecasting of machine health condition using arma/garch model. *Mechanical Systems and Signal Processing*, *24*(2), 546–558.

Scotti, C., & Benediktsdottir, S. 2009. Exchange rates dependence: what drives it? *FRB International Finance Discussion Paper*(969).

Sklar, M. 1959. Fonctions de répartition an dimensions et leurs marges, 8 pp. *Univ. Paris, Paris*.

Spagnolo, F., Psaradakis, Z., & Sola, M. 2005. Testing the unbiased forward exchange rate hypothesis using a markov switching model and instrumental variables. *Journal of Applied Econometrics*, *20*(3), 423–437.

Trivedi, P. K., Zimmer, D. M., et al. 2007. Copula modeling: an introduction for practitioners. *Foundations and Trends® in Econometrics*, *1*(1), 1–111.

Yang, L., Cai, X. J., Zhang, H., & Hamori, S. 2016. Interdependence of foreign exchange markets: A wavelet coherence analysis. *Economic Modelling*, *55*, 6–14.