A Monte Carlo Analysis of Seven Dichotomous Variable Confidence Interval Equations

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A MONTE CARLO ANALYSIS OF SEVEN DICHOTOMOUS VARIABLE CONFIDENCE INTERVAL EQUATIONS

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ABSTRACT

A MONTE CARLO ANALYSIS OF SEVEN DICHOTOMOUS VARIABLE CONFIDENCE INTERVAL EQUATIONS

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There are two options to estimate a range of likely values for the population mean of a continuous variable: one for when the population standard deviation is known and another for when the population standard deviation is unknown. There are seven proposed equations to calculate the confidence interval for the population mean of a dichotomous variable: normal approximation interval, Wilson interval, Jeffreys interval, Clopper-Pearson, Agresti-Coull, arcsine transformation, and logit transformation. In this study, I compared the percent effectiveness of each equation using a Monte Carlo analysis and the interval range over a range of population means to determine the accuracy of the equations. Results indicated that the Agresti-Coull equation and Clopper-Pearson equation are the most successful at locating the population proportion at least 95% of the time across the range of population proportions and that the Agresti-Coull equation has the narrower interval range.

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Introduction

A confidence interval is a range of scores that has a researcher-defined probability (commonly 95%) of containing the location of an unknown parameter. Confidence intervals can be computed for almost any sample statistic. Confidence intervals for sample means are common and are covered in almost every introductory presentation of statistics. A confidence interval for a mean can be computed for continuous variables such as height, age, or task completion time, as well as dichotomous variables such as yes/no or pass /fail. The focus of this study will be on the confidence interval equations for the means of dichotomous variables such as win/lose (e.g., in legal proceedings), pass/fail, and graduate/did not graduate.

Researchers and practitioners across all disciplines work with dichotomous variables. The ability to estimate the population mean of a dichotomous variable (i.e., a proportion) from a sample statistic can be used to estimate the spread of a virus or illness throughout a geographical population (infected versus not infected). Other uses worthy of note could be the success of drug trials in medical testing and percentage of produced product that will fail within a given timeframe (Wallis, 2013). In the field of Industrial-Organizational Psychology, there are several opportunities for requiring a mean of a dichotomous variable. Organization selection processes often have several fail-out points that may require examination if the organization is having difficulty hiring. The rate at which employees pass training interventions, as well as turnover are other dichotomous variables in the organizational setting.

The equation to estimate the population mean of a continuous variable has only two forms: one for when the population standard deviation is known and the other for when the population standard deviation is unknown. To date, seven equations have been proposed to create the confidence interval for a dichotomous variable: the normal approximation interval, the

Wilson score interval, the Jeffreys interval, the Clopper-Pearson interval, the Agresti-Coull interval, the Arcsine transformation, and the logit transformation (Brown et al., 2001). This study investigates the effectiveness of these seven equations at estimating the value of the population proportion.

Samples vs Populations

A population is the entirety of a group with one or more criterion of interest in common (Glen, 2013). Any group taken from that population for measurement and experiment purposes is known as a sample (Glen, 2013). All sample statistics are affected by sampling error, and thus do not accurately reflect the population value. Sampling error is the difference between the sample statistic used to estimate the population parameter and the unknown actual value of the parameter. Sampling error may be increased by having an insufficient sample size or a sample that was not collected via a probability sampling technique (Pedhazur & Schmelkin, 1991). Sources of sample bias include survivorship bias, convenience sampling, self-selection bias, and exclusion bias. Confidence intervals indicate that if the population is repeatedly sampled (with a probability sampling technique), using the same sample size and methods, the confidence interval calculated for the sample will include the population value a certain percentage of the time (again, commonly 95%).

Normal Approximation Interval

The normal approximation interval, also known as the Wald interval, is the most commonly used equation for creating the confidence interval for dichotomous variables. The equation itself is simple and makes use of the normal distribution.

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{N}} \qquad (1)$$

Where *N* is the total sample size, $\hat{p}$ is the sample proportion of successes, and *z* is the value from the z-table that corresponds to the desired confidence (e.g., 1.96 for a 95% confidence interval).

The normal approximation interval has the benefit of creating intervals that look like those from the continuous variable confidence interval equation (Newcombe, 1998). The equation is simple and creates what looks like an ideal confidence interval that follows the normal distribution pattern of the continuous variable confidence interval. There are, however, there are some serious issues with using the normal approximation interval (Brown et al., 2001; Newcombe, 1998; Wallis, 2013). One issue is that the equation can produce results that are out of range; the lower limit can be calculated as less than zero or greater than one (Newcombe, 1998; Wallis, 2013). Another issue is degeneracy, a zero-width interval that occurs when the sample proportion equals 0 or 1 (Brown et al., 2001; Cai, 2005; Newcombe, 1998; Wallis, 2013). The coverage of the equation is also shown to be an issue, due to the fact that differing *N* values can cause substantial change in how close the interval is to the desired level of confidence (Brown et al., 2001; Cai, 2005; Sauro & Lewis, 2005). A continuity correction equation can be applied to correct for the coverage issue and the degeneracy, but its use increases the chances of producing out of bounds results (Newcombe, 1998).

Wilson Score Interval

The Wilson score interval, as extended by Agresti and Couell (1998), is based on Wilson's (1927) work. Wilson proposed adding an additional two successes and two fails to the observed sample size to reduce the chances of the true mean being outside of the calculated interval (Sauro & Lewis, 2005; Wilson, 1927). The Wilson score interval is more cumbersome than the normal approximation interval.

$$p \approx (w^-, w^+) = \frac{1}{1+\frac{z^2}{N}}\left(\hat{p} + \frac{z^2}{2N}\right) \pm \frac{z}{1+\frac{z^2}{N}}\sqrt{\frac{\hat{p}(1-\hat{p})}{N} + \frac{z^2}{4N^2}} \qquad (2)$$

Where *z* remains the value from the z-table that corresponds to the desired confidence (i.e., 1.96 for a 95% confidence interval).

The addition of the two successes and two failures improves the coverage issue seen in the normal approximation interval towards the middle of the curve. One problem that remains is that the Wilson score interval fails to fully resolve the coverage consistency issues with the differing *N* values causing change in the closeness of the interval to the desired level of confidence as the numerator approaches 1 and 0 (Brown et al., 2001; Sauro & Lewis, 2005; Thulin, 2014; Wallis, 2013).

Jeffreys Interval

The Jeffreys interval is a Bayesian beta interval for *p*. The equations for the upper and lower bounds are as follows.

$$\mathrm{CI}_{\mathrm{J}}^{\mathrm{u}} = [B_{1-\frac{\alpha}{2}, X+1/2, N-X+1/2}] \tag{3}$$

$$\mathrm{CI}_{\mathrm{J}}^{\mathrm{l}} = [B_{\frac{\alpha}{2}, X+1/2, N-X+1/2}] \tag{4}$$

Where *X* is the number of successes, *N* is the total sample size, and 1-α is the equal-tailed probability interval.

The primary concern with the use of the Jeffreys interval is that it is derived from the noninformative Jeffreys prior, another Bayesian beta distribution, which means that the exact endpoints for each of the intervals requires statistical software, such as SAS, to compute (Brown et al., 2001; Thulin, 2014). The Jeffreys interval requires no continuity correction equation and improves the coverage issues of both the normal approximation interval and the Wilson score interval (Cai, 2005; Thulin, 2014). One distinct advantage of the Jeffreys interval is that it is equal tailed, the probability of the interval being above or below the true value of the population mean is 2.5% (Brown et al., 2001; Thulin, 2014).

Clopper-Pearson Interval

The Clopper-Pearson interval is also known as the “exact” interval and considered the gold standard (Agresti & Min, 2001; Brown et al., 2001; Newcombe, 1998; Reiczigel, 2003; Ross, 2003; Sauro & Lewis, 2005; Thulin, 2014). The equation itself is written as

$$\mathrm{CI}_{\mathrm{CP}}^{\mathrm{L}} = [B_{1-\frac{\alpha}{2},X,N-X+1}] \quad (5)$$

$$\mathrm{CI}_{\mathrm{CP}}^{\mathrm{U}} = [B_{\frac{\alpha}{2},X+1,N-X}] \quad (6)$$

Where *x* is the number of successes, *n* is the total sample size, $\mathrm{Bin}(n;\theta)$ is a binomial random variable with *n* trials and probability of success θ, and $\frac{\alpha}{2}$ is the significance.

The variation of the parameters using an *F* distribution rather than a beta distribution to be used in this thesis is as follows:

$$\mathrm{CI}_{\mathrm{CP}}^{\mathrm{L}} = [1 + \frac{n-x+1}{xF_{2x,2(n-x+1),1-\frac{\alpha}{2}}}]^{-1} \quad (7)$$

$$\mathrm{CI}_{\mathrm{CP}}^{\mathrm{U}} = [1 + \frac{n-x}{(x+1)F_{2(x+1),2(n-x),\frac{\alpha}{2}}}]^{-1} \quad (8)$$

The interval is an inversion of the equal-tailed binomial test and based directly on binomial distribution (Clopper & Pearson, 1934; Reiczigel, 2003; Thulin, 2014). The Clopper-Pearson interval is also the most conservative of the intervals, used with the belief that overestimation is better than underestimation, giving it a wide but well-defined coverage area (Brown et al., 2001; Clopper & Pearson, 1934; Sauro & Lewis, 2005; Thulin, 2014). The conservative nature of the interval is not without criticism, particularly regarding smaller sample sizes and the perceived lack of practicality due to computational difficulty (Brown et al., 2001; Newcombe, 1998; Sauro & Lewis, 2005; Thulin, 2014).

Agresti-Coull Interval

The Agresti-Coull interval formally adds the additional two successes and two failures into the normal approximation interval, eliminating the need for a continuity correction equation while maintaining a simple equation (Agresti & Coull, 1998; Brown et al., 2001).

$$\tilde{p} \pm z\sqrt{\tilde{p}(1-\tilde{p})/\tilde{n}} \quad (9)$$

Where $\tilde{n} = n + z^2$ and $\tilde{p} = (X + \frac{z^2}{2})/\tilde{n}$, where *n* is the total sample size, *X* is the number of successes, and *z* is the value from a z-table, 1.96 for a 95% confidence interval. Because $z = 1.96$ and $z^2/2 = 1.92$, which is approximately 2 and $z^2 = 3.8416$, which is approximately 4, this adjustment is often referred to as adding two successes and two failures.

The maintenance of a simple equation with the addition of the two successes and two failures is accomplished by substituting $\tilde{p}$ in place of $\hat{p}$ in the equation. The value $\tilde{p}$ is the weighted average, and the Wilson score interval midpoint. Using the weighted average with weight given to $\hat{p}$ shrinks the midpoint towards .5. As sample size increases, the rate of shrinkage decreases (Agresti & Coull, 1998; Brown et al., 2001). The Agresti-Coull also has the advantage over the normal approximation interval of being more consistent with smaller sample sizes (Thulin, 2014).

Arcsine Transformation

The arcsine transformation is an equation used to stabilize variance by mathematically pulling the ends of the interval outwards. This is used primarily with proportion data.

$$\sin^2\left(\arcsin\left(\sqrt{p}\right) - \frac{z}{2\sqrt{n}}\right) < \theta < \sin^2\left(\arcsin\left(\sqrt{p}\right) + \frac{z}{2\sqrt{n}}\right) \quad (10)$$

Like the normal approximation interval, the arcsine transformation has significant coverage issues towards the 0 and 1 boundaries (Brown et al., 2001). The arcsine transformation also has demonstrated consistency issues, despite ease of computation (Warton & Hui, 2011).

Logit Transformation

The logit transformation is used to convert proportional data to an approximate normal distribution (Brown et al., 2001; Cramer, 2003). The equation below is the generalized version.

$$t_a = \log\left(\frac{p^a}{(1-p)^{2-a}}\right) \quad (11)$$

The only value of a that is relevant for the purpose of this thesis is $a = 1$. When $a = 1$, the equation is able to be used to transform a proportional data into an approximately normal distribution.

The specific logit transformation for calculating confidence intervals for dichotomous variables is shown below and is an inversion of the odds for a normal approximation interval.

$$\hat{\lambda} = \ln\left(\frac{\hat{p}}{1-\hat{p}}\right) \quad (12)$$

The confidence interval itself is

$$CI(\lambda) = [\lambda_l, \lambda_u] = [\hat{\lambda} - \kappa\hat{V}^{1/2}, \hat{\lambda} + \kappa\hat{V}^{1/2}] \quad (13)$$

Where $\hat{V} = \frac{n}{X(n-x)}$ and κ functions as the desired confidence at $100(1-\alpha)\%$ using the values obtained from a z-table.

The logit interval for p is as follows and is the inversion of the interval (13):

$$\left[\frac{e^{\lambda_l}}{1+e^{\lambda_l}}\right] < p < \left[\frac{e^{\lambda_u}}{1+e^{\lambda_u}}\right] \quad (14)$$

The logit transformation is the transformation that is being brought up as the replacement for the arcsine transformation due to advances in technology making computation less difficult (Warton & Hui, 2011). The primary criticism of the logit transformation is that the interval is wider than even the Clopper-Pearson interval (Brown et al., 2001). Due to the nature of the natural logarithmic transformation, the Logit interval cannot be computed when the sample proportion equals 0 or 1.

The Present Study

The present study used a Monte Carlo analysis to determine the conditions under which the above seven binomial confidence interval equations provide the most accurate estimation of the population mean. Accuracy in this study is defined as the smallest interval range containing the population mean with the desired frequency (e.g., 95% of the time for a 95% confidence interval). The smaller the interval range combined with a higher frequency of occurrence of the population mean demonstrated in this study where it is known indicates that the equation may be more likely to contain the population mean when it is unknown. A Monte Carlo analysis is a technique that uses large, computer-generated data sets to test mathematical equations to determine their effectiveness through a comparison of their results to known parameters. A Monte Carlo analysis allows researchers to test how effective a statistical method is in a variety of different conditions over repeated and randomized large samples that are difficult or impossible to obtain otherwise.

This is an exploratory study and expands primarily on the work of Brown, et al. (2001). Brown, et al. (2001) varied the sample size for each of the equations rather than the population mean to determine the coverage probability of the equations. Maintaining the sample size and varying the population mean allows for the comparison of the frequency that the confidence interval contains the population mean and the range of the interval. The comparison will show if one of the equations is more appropriate for use.

Method

Conditions

A Monte Carlo analysis was used to examine the effectiveness of the seven binomial confidence interval equations. The dependent variable was whether the confidence interval formed by a given equation included the known population proportion. A 95% confidence interval should include the population proportion in 95% of the cases. Sample size was kept constant at 100 for all conditions. Only the population proportion was varied. The population means that were tested ranged between .01 and .99 in increments of approximately .05.

Procedure

For each condition sample of 100 cases were created from a population with a specified proportion. The sample proportion was computed from these 100 data points. The seven equations were used to construct the confidence intervals around the sample proportions. For each confidence interval, the dependent variable was coded as 1 if the interval contained the population proportion and zero if it did not. This procedure was repeated 10,000 times and results were averaged to yield the percent of cases in which the population mean was correctly located by the confidence interval for each of the seven equations.

Results

Table 1 lists the interval range (upper value minus lower value) for a sample of $N = 100$ for each of the seven confidence interval equations at the various population proportions. The ranges show the width of the confidence intervals. Table 2 lists the effectiveness of each equation across 10,000 samples ($N = 100$ for each sample) at the various population proportions. Each entry in Table 2 is the percent of samples for which a given interval contained the population proportion.

Inspection of Tables 1 and 2 offers some information regarding the accuracy of the binomial confidence interval equations. The Clopper-Pearson equation was consistently the most effective equation, particularly at the extremes with a percent effectiveness of 98.33 at .01 and 98.13 at .99. That accuracy came at a price: it was also the equation with the widest interval range across most population proportions. The Agresti-Coull interval equation and Jeffreys interval equation were similar in effectiveness behind the Clopper-Pearson equation. The Agresti-Coull interval equation was more effective closer to the extremes with a higher percent effectiveness at both .05 (96.75) and .95 (96.62). Both equations had narrower interval ranges than the Clopper-Pearson. The Jeffreys interval equation had the narrower interval range of the two. The logit transformation was the next most effective equation due to differences at .01 (97.39) and .99 (96.99). Otherwise, the logit transformation is as effective as the Agresti-Coull and Jeffreys interval equations and the interval range is only slightly larger. The logit transformation suffers from one serious drawback: it can't be computed when the sample proportion is 0.0 or 1.0. Although such sample proportions are rare, they occurred almost a third of the time for population proportions of .01 and .99 and approximate half a percent of the time for population proportions of .05 and .95. The Wilson interval equation also has an effectiveness greater than 90% across all conditions. The Wilson interval, however, is less effective (92% versus 95%) at extreme population proportions (.01 and .99). The arcsine and Normal intervals are the least effective at the extremes (less than 65% at .01 and .99). The arcsine transformation and normal approximation also have similar interval ranges.

In summary, the only equations that successfully locate the population proportion at least 95% of the time across the entire range of population proportions are Clopper-Pearson and Agresti-Coull. Jeffreys comes close but dips to 93% in a few instances (e.g., population proportion = .10). Between Agresti-Coull and Clopper-Pearson, Agresti-Coull has the narrower interval, making it a slightly better choice.

Discussion

The purpose of this study was to evaluate the accuracy of the seven proposed dichotomous variable confidence interval equations: the normal approximation interval equation, the Wilson score interval equation, the Jeffreys interval equation, the Clopper-Pearson equation, the Agresti-Coull equation, the arcsine transformation, and the logit transformation at various population proportions. Industrial-organizational psychologists work with several dichotomous variables, including selection ratios, turnover, and pass rates for employee training interventions. Mean-based confidence intervals play a large role in research and applied practice. Although the procedures for computing a confidence interval for the mean of a continuous variable are well understood, less is known about confidence for the mean of a dichotomous variable. Accurately defining the likely location of the population proportion from a sample proportion allows for better decision making in both research and applied practice.

As expected from the review of the literature, the Clopper-Pearson equation performed well but also had the largest interval range. The Agresti-Coull equation displayed similar accuracy and featured slightly narrower intervals. As a bonus, Agresti-Coull is one of the easier equations to compute, as it is a simple modification (add two successes and two failures to the sample statistics) to the basic normal approximation interval. With Agresti-Coull there is no need for a beta distribution (Clopper-Pearson and Jeffreys), mathematic or trigonometric functions (arcsine and logit), or long, convoluted equations (Wilson).

Conclusion

Researchers and practitioners need guidance for estimating the population proportion when a dichotomous variable is involved. It is necessary to have the most accurate estimate of the true population proportion to make sound research and business decisions, such as adjusting procedures regarding the selection and turnover of employees. The results of this study indicate that it would be beneficial for scholars and practitioners to utilize the simple yet effective Agresti-Coull equation for the most accurate estimates of population parameters.

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Table 1

Interval Range (Upper Minus Lower) for Given Proportions

Proportion	Normal Interval	Wilson Interval	Jeffreys Interval	Clopper-Pearson	Agresti-Coull	Arcsine Transform	Logit Transform
.01	.039	.053	.045	.054	.064	.039	.066
.05	.085	.090	.087	.096	.096	.085	.094
.10	.118	.119	.118	.127	.123	.117	.121
.15	.140	.140	.139	.149	.142	.139	.142
.20	.157	.155	.155	.165	.157	.156	.157
.25	.170	.168	.168	.178	.169	.169	.169
.30	.180	.177	.178	.187	.177	.178	.178
.35	.187	.184	.185	.195	.184	.186	.185
.40	.192	.189	.190	.199	.189	.191	.190
.45	.195	.191	.193	.202	.191	.194	.193
.50	.196	.192	.194	.203	.192	.195	.194
.55	.195	.191	.193	.202	.191	.194	.193
.60	.192	.189	.190	.199	.189	.191	.190
.65	.187	.184	.185	.195	.184	.186	.185
.70	.180	.176	.178	.187	.177	.178	.178
.75	.170	.168	.168	.178	.169	.169	.169
.80	.157	.155	.155	.165	.157	.156	.157
.85	.140	.140	.139	.149	.142	.139	.142
.90	.118	.119	.118	.127	.123	.117	.121
.95	.085	.090	.087	.096	.096	.085	.094
.99	.039	.053	.045	.054	.064	.039	.066

Note. $N = 100$ for interval width computations.

Table 2

Binomial Confidence Interval Effectiveness at Varying Population Proportions

Proportion	Normal Interval	Wilson Interval	Jeffreys Interval	Clopper-Pearson	Agresti-Coull	Arcsine Transform	Logit Transform
.01	64.06	91.95	98.33	98.33	98.33	62.42	97.39
.05	88.08	96.75	93.80	98.49	96.75	95.54	97.27
.10	93.13	93.59	95.41	95.41	96.97	95.41	95.15
.15	93.51	93.54	95.17	96.51	96.51	95.17	96.51
.20	93.73	94.04	95.76	96.95	94.04	95.76	94.04
.25	94.47	94.80	94.80	96.12	94.80	93.15	94.80
.30	95.18	93.84	95.18	96.49	95.15	95.18	96.49
.35	94.12	95.42	95.42	95.42	95.42	95.42	95.42
.40	95.10	95.10	95.10	96.08	95.10	95.10	95.10
.45	94.48	94.48	94.48	96.42	94.48	94.48	94.48
.50	93.83	93.83	93.83	96.04	93.83	93.83	93.83
.55	94.60	94.60	94.60	96.59	94.60	94.60	94.60
.60	94.69	94.69	94.69	95.83	94.69	94.69	94.69
.65	94.03	95.23	95.23	95.23	95.23	95.23	95.23
.70	95.20	93.92	95.20	96.36	95.08	95.20	96.36
.75	94.89	95.36	95.36	96.47	95.36	93.78	95.36
.80	93.46	94.29	95.50	96.73	94.29	95.50	94.29
.85	93.32	93.14	94.63	96.20	96.20	94.63	96.20
.90	93.27	93.39	95.34	95.34	96.92	95.34	94.97
.95	87.51	96.62	93.34	98.39	96.62	95.11	97.11
.99	62.23	92.41	98.13	98.13	98.13	60.40	96.99

Note. $N = 100$ for each sample. Each table entry represents results (percent of cases in which population proportion falls within specified confidence interval) across 10,000 samples. For population proportion values .01, .05, .90, .95, and .99 the logit transformation confidence interval was not able to be calculated the total $N = 10{,}000$. For population proportion = .01, $N = 6409$. For population proportion = .05, $N = 9946$. For population proportion = .90, $N = 9999$. For population proportion = .95, $N = 9949$. For population proportion = .99, $N = 6227$.